Hint 2

IMPORTANT! The next task is both a hint and an alternative to the main task. Three important points:

1. You can continue to send the solution to the main problem.
2. At any moment before the final deadline you can start to solve the Alternative problem. If you do so, at the beginning of the solution write: *I am doing the Alternative problem!* In this case a penalty coefficient for the Alternative problem is

$$0{,}7 \cdot \sum_i \frac{k_i \cdot p_i}{10},$$

where p_i is a point for the problem item, and k_i is a penalty coefficient for the corresponding problem's item at the moment of moving to the Alternative problem. In other words, maximal points for the alternative problem equals to the maximal points you can gain at the moment of moving to the alternative one multiplied by 0,7. Also, we remind you that a penalty coefficient can't be less than 0,1. Solutions of the main problems from that moment will not be checked. Be careful!

3. The task consists of several items. The penalty multiplier earned **before** is applied to all points. In the future, each item is evaluated as a separate task. If you send a solution without any item, this item's solution is considered as Incorrect. For more information about scoring points for composite tasks, see the rules of the Cup.

Alternative problem

Part 1. Phase portraits

1. *(0.5 points)* Material point of unit mass is at rest. Constant force F begins to act on the point. Plot a phase portrait of this motion for six different values of the force F.
2. *(1.0 point)* A charged particle moves in the field of a uniformly charged plane. The charge of the plane and the charge of the particle have the same sign. If particle reaches this plane, it goes through it. Plot all possible types of phase portraits for the system under different initial conditions.

Part 2. Two stars

Consider a system consisting of two stars of the same mass moving around a common barycenter in circles with constant speed.

3. *(0.5 points)* Prove that the circles along which the stars move are identical..

Consider a conditional system of units. Assume that stars have mass $m = 1/2$, orbital period 2π, radius of circular orbits of stars is $a = 1$, gravitational constant $G = 1$. A planet moves in space with a double star, the mass of planet is so smaller than masses of the stars that its wagging on their motion can be neglected. Stars interact with the planet according to the law of universal gravitation. Consider the case when the planet moves along a straight line passing through the common center of mass of the stars and orthogonal to the plane of their motion: $x \equiv 0, y \equiv 0, z = z(t)$.

4. *(1.5 points)* Write a differential equation for the variable z and draw a phase portrait on a plane $z, \dot{z}$.

Let the stars move along elliptical trajectories about a common center of mass with an eccentricity e. Then the equations of motion become more complicated:

$$\ddot{z} = -\frac{z}{\left((1 - e\cos E)^2 + z^2\right)^{3/2}}, \quad E - e\sin E = t.$$

5. *(1.5 points)* Plot the Poincare section on the $z, \dot{z}$ plane for various eccentricities: $e = 0$, 0.1, 0.3, 0.5, 0. Take $t = 2\pi$ as the mapping step, initial conditions $0 \leq z(0) \leq 3$, $\dot{z}(0) = 0$, $E = 0$.
6. *(3.0 points)*Specify the initial conditions for which the following modes of motion are realized::
   (a) the planet is at rest in the barycenter of the binary system (Euler's solution);
   (b) damped oscillations near the center of mass;
   (c) periodic oscillations;
   (d) oscillations with increasing amplitude, ending with the departure of the planet to infinity;
   (e) oscillations with increasing amplitude, but without reaching to infinity (oscillating motions).

Part 3. Path of the Point. Linear case II

Follow the link to the alternative notebook. Review in the notebook the cell «Path of the Point». The following «function» is analyzed in the section: $\{\lambda x\}$ — the fractional part of λx, where $\lambda = \text{const} > 0$. In all parts of this section of the problem, we will only consider the values of $x \in [0, 1]$.

7. *(0.5 points)* Draw $f(x)$ for $\lambda = 2$.
8. *(0.5 points)* Qualitatively plot $f^2(x)$ and $f^4(x)$ for values of the parameter $\lambda = 2$, $\lambda = 2{,}5$, $\lambda = 4$.
9. *(0.5 points)* Find the dependence of the number of equilibrium positions on the degree of the function for $\lambda = 2$.
10. *(0.5 points)* Open the notebook cell in the section «Discrete model. Linear case». Change the «function» to the one that is being studied in the alternative problem. This section shows all the values that are obtained after a large number of mappings with λ from 10 to 20. Try to analyze the result.