LPR VI Cup

9.s06.e01

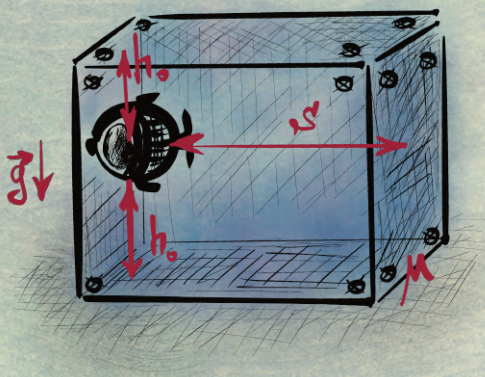
"Who said that the crayfish walks backwards? It's just going its own way."
Mikhail Mikhailovich Mamchich

Ant-Voyageur and the Bathyscaphe

Traveller-Ant, having explored all the available continent, decided to venture beyond its borders. By a happy coincidence, our hero, while having breakfast with croissant crumbs and drops of cappuccino on a café tablecloth in Montmartre and reading fresh (as the croissant) issue of *Elle* magazine, noticed an intriguing article. It reported that British scientists had recently discovered an unknown object in the ocean, whose size and speed exceeded those of a whale, and which itself closely resembled a giant narwhal. Immediately after reading it, our active hitchhiker decided: whatever it is, he must befriend it and, perhaps, if it doesn't mind (which has never happened before), hitch a ride to the neighboring continent.

For this absolutely random, unplanned, unexpected, intriguing, underwater ~~methodical meeting~~ rendezvous to actually happen, Ant began searching for a solution to his ablutophobia and other, not so significant, obstacles such as the lack of gills and fins. Setting out to overcome his fear, he started looking for suitable equipment. In the "fbuy-sell-trade" column, next to the ads "garage for sale" and "will make a door," our hero found a small note about a used bathyscaphe "Denise" for sale. Having obtained the necessary sum and completed the purchase, he quickly realized—staying on the continent was no longer an option for him, and he urgently needed to learn how to use his newly acquired purchase.

For this purpose, he placed himself inside the bathyscaphe and the bathyscaphe inside a fully enclosed aquarium shaped like a rectangular prism, filled to the brim with a liquid of density ρ_{liq}. The aquarium rests on a horizontal surface with a friction coefficient μ. The masses of the liquid and the aquarium's frame are M_{liq} and M_{frame}, respectively. The bathyscaphe has a volume V, an average density (including Ant) of ρ_{ant}, and is positioned near the center of the left wall of the container.



The height of the container's is $2h_0 + h_{\text{b}}$, and the length is $S + S_{\text{b}}$, where h_{b} and S_{b} are the dimensions of the bathyscaphe [1].Assume that in all sections, the bathyscaphe's dimensions are not negligible.

The "Denise" came with instructions in Chinese, so its mechanism remained a mystery both to Ant and to you. In all sections, Ant's velocity is given relative to the stationary horizontal surface. After each experiment, Ant resets the system to its initial state (all velocities zero,

[1]The distance from the highest point of the bathyscaphe to the top of the aquarium and from its lowest point to the bottom is h_0. The distance from its rightmost point to the opposite wall is S.

positions unchanged). The liquid is incompressible, and turbulent effects are strongly recommended to be neglected. In all sections, motion occurs in a plane parallel to the aquarium wall closest to the reader.

Partie Un: A Horizontal Case

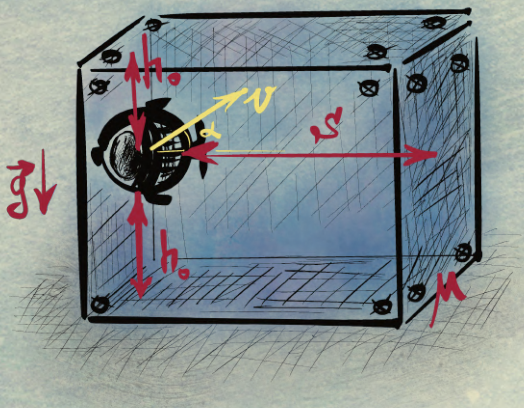
First, Ant decided to learn how to swim horizontally. After pressing a few buttons randomly, he felt he had succeeded—the bathyscaphe abruptly started moving at speed v toward the opposite wall of the container (see figure). Assuming the velocity vector remains constant during motion, find:

1. *(0.5 points)* The magnitude and direction of the aquarium's velocity immediately after the bathyscaphe's launch.
2. *(0.5 points)* The minimum length S_{min} if it stops **before** the "Denise" reaches the opposite wall.
3. *(1 point)* Assuming length S is $2S_{\text{min}}$, determine the work done by pressure forces on the aquarium's walls during the bathyscaphe's motion from one side to the other.

Assume that the values h_0, M_{liq}, M_{frame}, V, ρ_{liq}, ρ_{ant}, μ, V, and v are known.

Partie deux: Angle Case

Having learned to direct the velocity vector horizontally, Ant continued figuring out the bathyscaphe's controls. In the next experiment, he managed to make the "Denise" move with constant speed and direction, reaching exactly the midpoint of the top edge of the opposite wall. The velocity vector forms an angle $\alpha = 46°$ with the horizontal.

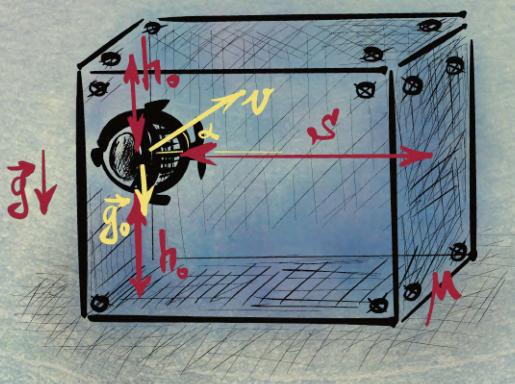


1. *(3 points)* What is the coefficient of friction between the aquarium and the table in this part?

For this part, assume: $h_0 = 1$ m, length $S = 1$ m, initial velocity $v = 1$ m/s, mass of the aquarium frame $M_{\text{frame}} = 200$ g, mass of the liquid $M_{\text{liq}} = 600$ g, liquid density $\rho_{\text{liq}} = 1000$ kg/m^3, average density of *Denise* with Ant $\rho_{\text{ant}} = 1600$ kg/m^3, volume of the bathyscaphe with Ant $V = 200$ mL, gravitational acceleration g $= 10$ m/s^2.

Partie trois: A Ballistic Case

Ant continued his training and in the next launch managed to direct the bathyscaphe's initial velocity v at an angle α ($0 < \alpha < 90°$) to the horizontal, while controlling it during motion so that it moved with constant acceleration g_0 directed vertically downward.



1. *(5 points)* Find numerical values for the aquarium's velocity immediately after Ant's first collision with the wall for angles $\alpha_1 = 5°$, $\alpha_2 = 10°$, $\alpha_3 = 20°$, $\alpha_4 = 40°$.

When colliding with the aquarium wall, Ant rebounds and reverses the velocity component perpendicular to the wall while keeping the parallel component unchanged.

For this part, assume: initial velocity $v = 2$ m/s, mass of the aquarium frame $M_{\text{frame}} = 200$ g, mass of the liquid $M_{\text{liq}} = 600$ g, liquid density $\rho_{\text{liq}} = 1000$ kg/m^3, average density of *Denise* with Ant $\rho_{\text{ant}} = 1400$ kg/m^3, coefficient of friction $\mu = 0.02$, volume of the bathyscaphe with Ant $V = 200$ mL, dimensions $S = 30$ cm and $h_0 = 5$ cm, gravitational acceleration $g = 10$ m/s^2, Ant's acceleration $g_0 = 9$ m/s^2.

P.S. Yes, this is that same Ant.

First Hint — 28.04.2025 20:00 (Moscow time)
Second Hint — 30.04.2025 12:00 (Moscow time)

Final of the First Episode — 02.05.2025 20:00 (Moscow time)