LPR VI Cup

9.s06.e03

Hint 2

IMPORTANT! The next task is both a hint and an alternative to the main task. Three important points:

1. You can continue to send the solution to the main problem.
2. At any moment before the final deadline you can start to solve the Alternative problem. If you do so, at the beginning of the solution write: *I am doing the Alternative problem!* In this case a penalty coefficient for the Alternative problem is

$$0{,}7 \cdot \sum_i \frac{k_i \cdot p_i}{10},$$

   where p_i is a point for the problem item, and k_i is a penalty coefficient for the corresponding problem's item at the moment of moving to the Alternative problem. In other words, maximal points for the alternative problem equals to the maximal points you can gain at the moment of moving to the alternative one multiplied by 0,7. Also, we remind you that a penalty coefficient can't be less than 0,1. Solutions of the main problems from that moment will not be checked. Be careful!
3. The task consists of several items. The penalty multiplier earned **before** is applied to all points. In the future, each item is evaluated as a separate task. If you send a solution without any item, this item's solution is considered as Incorrect. For more information about scoring points for composite tasks, see the rules of the Cup. **Since switching to an alternative selection, there is no opportunity to return to solving the main task.** Also, after switching to an alternative task **the points for the main task are reset**.

Alternative task

It seemed as if this place had been created for Hidden. He took a shiny flask out of his travel bag, took a small sip, and felt a tart yet pleasant taste in his mouth. The evening breeze brushed against Hidden's face, while the silence seemed to settle over this City like a lulling dome.

Throwing back the hood of his newly purchased cloak, he walked along the deserted streets, regretting that there was no familiar crowd around him — one in which he could always easily disappear. Hidden knew exactly how much time had passed since his arrival in the City. He flipped open the lid of his pocket watch to make sure he would be precisely on time at the appointed place. The watch was the latest model, equipped with a built-in compass (as well as a barometer and a host of other instruments).

After checking his direction, he moved toward the avenue and saw a tall, beautiful building, its Ionic columns supporting a shingled roof adorned with an intriguing bas-relief at the top. Hidden approached the heavy door and, with some effort, pushed it open to enter.

Inside, it was surprisingly warm, though it seemed the stone walls should have given off a chill. Hidden walked to the center of the large hall, along the left wall of which stood tall bookcases reaching up to the ceiling, filled with books in green bindings decorated with gold embossing, while on the right hung beautiful tapestries depicting sleeping lions, grazing bulls, and birds soaring in the sky. Hidden smiled when he saw, deep in the room, a mirror with which he had always had an excellent relationship.

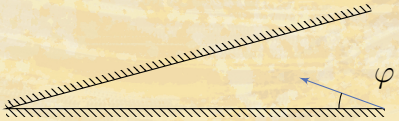
The heavy door behind him slammed shut with a resounding thud, breaking the silence of the sleeping City. Hidden turned around unhurriedly: before him stood a woman with a leonine bearing and an aquiline profile. Her unblinking eyes, devoid of eyelashes, stared piercingly at him. On her left arm dozed a hairless cat, which she absentmindedly stroked with slender fingers.

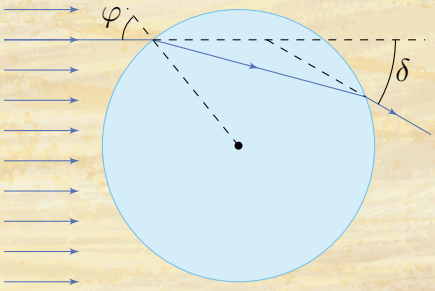
"You will stay here and never hide again. Answer correctly, and you will have the chance to leave," she said.

First riddle

Two flat mirrors OA and OB form a dihedral angle $4AOB$, which equals to $15°$, $OA = OB = 10$ m. A ray emerges from point B in AOB plane, at an angle $\varphi = 20°$ to OB.

1. *(2 points)* How many times will the ray reflect off the mirrors?





A sphere made of an optically transparent material is placed in a parallel beam of light (see figure). The angle of incidence of one of the rays is $\varphi = \operatorname{arctg}(4/3)$. The angle by which

the ray is deviated from its original direction after two refractions at the sphere's surface is $\delta = 2\,\mathrm{arctg}(7/24)$.

2. *(0 points)* Find the refraction index.

Second riddle

Draw the positions of the Sun, the Earth and the Moon when following are witnessed from the Earth surface:

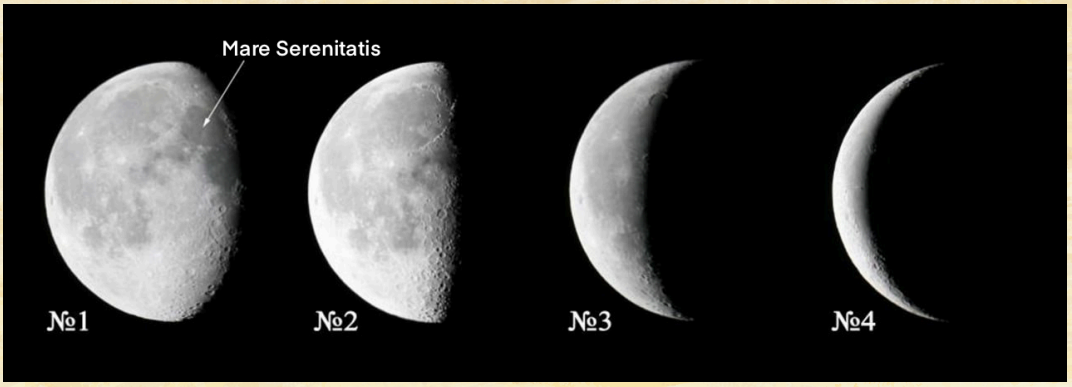
1. *(0 points)* a full moon (fully illuminated),
2. *(0 points)* a half moon,
3. *(0 points)* a completely dark moon.
4. *(1 point)* Is the moon always visible at night? Explain the answer and draw the necessary diagrams of celestial bodies.

The picture shows a photograph of an airplane against the background of the Moon.

5. *(1 point)* Estimate the distance between the photographer and the plane (the plane's wing span is around 25 meters)

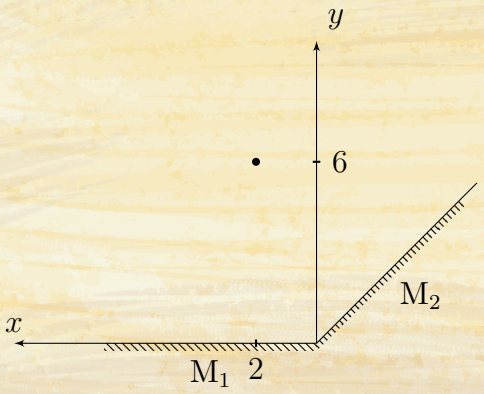
The photograph shows the phases of the Moon. The phase of the Moon, Φ, is defined as the ratio of the width b of the lunar crescent to the diameter d of the lunar disk.

6. *(0 points)* Using the photograph, determine the phase of the Moon in each image.
7. *(1 point)* Draw the approximate position of the Moon relative to the Earth at the moment the second photograph was taken. Assume the Moon's orbit is circular and lies in the plane of the ecliptic.
8. *(1 point)* Estimate the diameter of the Mare Serenitatis.



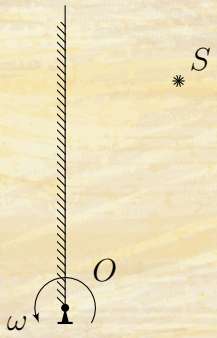
Third riddle

A point source is initially located at coordinates $(2; 6)$, above two mirrors intersecting at an angle $\alpha = 135°$.



1. *(1 point)* The source is moved vertically downward to the point with coordinates $(2; 0)$. Find the dependence of the number of images of the source in this system of mirrors on the y coordinate.
2. *(1 point)* The source is moved horizontally to the right to the point with coordinates $(-6; 6)$. Find the dependence of the number of images of the source in this system of mirrors on the x coordinate.
3. *(1 point)* The source is moved along a straight line to the point (0,0). Find the dependence of the number of images of the source in this system of mirrors on the coordinate.

The starting position of the source is the same in all three cases. The scale in the diagram can be used for reference.



A plane mirror rotates with an angular velocity ω around the axis O (see the figure).

4. *(1 point)* At what speed v does the image S' of point S move if the distance $OS = L$?

A useful resource for visualization (and for creating diagrams) is Ray Optics Simulation. It is important to remember that its illustrations cannot be used as a means of proof, but for making drawings and checking your solutions, this is a very helpful link. Make use of it!