LPR V Cup

9.s05.e02

Hint 2

IMPORTANT! The next task is both a hint and an alternative to the main task. Three important points:

1. You can continue to send the solution to the main problem.
2. At any moment before the final deadline you can start to solve the Alternative problem. If you do so, at the beginning of the solution write: *I am doing the Alternative problem!* In this case a penalty coefficient for the Alternative problem is

$$0{,}7 \cdot \sum_i \frac{k_i \cdot p_i}{10},$$

where p_i is a point for the problem item, and k_i is a penalty coefficient for the corresponding problem's item at the moment of moving to the Alternative problem. In other words, maximal points for the alternative problem equals to the maximal points you can gain at the moment of moving to the alternative one multiplied by 0,7. Also, we remind you that a penalty coefficient can't be less than 0,1. Solutions of the main problems from that moment will not be checked. Be careful!

3. The task consists of several items. The penalty multiplier earned **before** is applied to all points. In the future, each item is evaluated as a separate task. If you send a solution without any item, this item's solution is considered as Incorrect. For more information about scoring points for composite tasks, see the rules of the Cup. **Since switching to an alternative selection, there is no opportunity to return to solving the main task.** Also, after switching to an alternative task **the points for the main task are reset**.

Alternative task

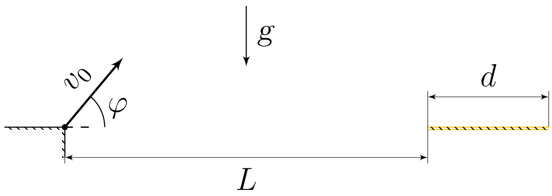
Part 1. Another Brick in the Sky

Hedgehog is playing a game with the seagulls: seagulls are flying at a height of H and Hedgehog throws ~~another~~ brick in the sky vertically upwards with a random initial velocity v_0 (all velocities are equiprobable). Since seagulls fly a lot, if at some point the brick reaches the height H, then Hedgehog wins, and one of the seagulls loses. Find the probability of Hedgehog's victory if:

1. *(1 point)* $m = 5.2$ kg, $v_0 \in [10,\ 15]$ m/s, $H = 10$ m, $g = 10$ m/c^2.
2. *(1 point)* $m = 5.2$ kg, $v_0 \in [15,\ 20]$ m/s, $H = 10$ m, $g = 10$ m/c^2.

Part 2. To ~~meet~~ hit one seagull with one stone

After a few marine sandglasses, the seagulls got tired of the previous game and suggested changing the rules.Hedgehog takes a ship's cannon from the gentlemen, and the seagull is positioned at the level of the cannon at a distance L from it. Hedgehog launches a projectile of mass m with a velocity v_0 at some angle to the horizon. Unfortunately, Hedgehog has paws, so the angle to the horizon lies in the range $(0, 13\pi)$. All angles are equally probable. If the projectile hits the seagull, she loses, and Hedgehog wins. Assuming the horizontal size of the seagull is d, find the probability with which ~~Hedgehog and the gentlemen will feed the fish~~ Hedgehog will win, if:

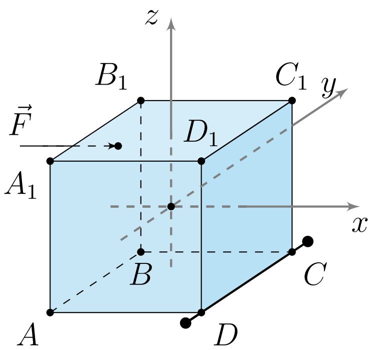


1. *(1.5 points)* $m = 12$ kg, $v_0 = 20$ m/s, $d = 1$ m, $L = 30$ m, $g = 10$ m/s^2.
2. *(1.5 points)* $m = 12$ kg, $v_0 = 20$ m/s, $d = 1$ m, $L = 39.5$ m, $g = 10$ m/s^2.

Consider projectile is a point.

Part 3. A hatch to the hold

Hedgehog was walking around the ship and discovered a very strange hatch in the hold. The hatch is in the form of a cube $ABCDA_1B_1C_1D_1$, with the side DC hinged in such a way that the hatch can freely rotate around it (see figure). The length of the hatch side is $2a$, and the mass of the hatch is m. The origin is located at the geometric center of the hatch. The center of mass of the hatch has known coordinates $(x,\ y,\ z)$, where $|x| < a$, $|y| < a$, $|z| < a$. Hedgehog applies a force $F = mg$ to a random point of a random face of the cube perpendicular to this face.



1. *(2 points)* Find the probability of the hatch moving.

The hold is located beneath the face $ABCD$, assuming that the hatch cannot open into the hold.

Part 4. Series method

Hedgehog, as an astronomer, knows that the energy of gravitational interaction between a spherical planet and a point mass m located outside the planet is equal to:

$$\mathfrak{E} = -\frac{GmM}{r}$$

1. *(0.5 points)* Using approximate formulas obtain an expression for the potential energy of a body at a distance $R + h$ from the center of the planet, where R is the radius of the planet, and h is the small height ($h \ll R$) to which the body was raised.

In addition, Hedgehog knows that in relativistic mechanics, the kinetic energy of a body is equal to:

$$E_K = \sqrt{m^2c^4 + p^2c^2} - mc^2$$

2. *(0.5 points)* Assuming that the speed of the body is much less than the speed of light, find the expression for classical kinetic energy.

Part 5. ~~The Seagull went home in the box~~ The box on the deck

The box is located on the horizontal and rough deck. The mass of the box ~~together with the seagull~~ is m. The coefficient of friction between the box and the deck is μ. Hedgehog applies a force F to the center of the box, directed at an angle α to the horizon. The angle is $\alpha = \arctan(\mu) + \delta$, где $|\delta| \ll 1$. The force F is minimal for this angle that must be applied to move the box from its place.

1. *(2 points)* Express F through μ, m, g, δ using approximate formulas to the first non-zero order.

The box has the shape of a rectangular parallelepiped. The vertical projection of the force F is directed against the acceleration of free fall g.

Since Hedgehog was an astronomer and was passionate about star maps, he knew several amusing and useful facts that might come in handy for you:

$$\frac{1}{1+\xi} \approx 1-\xi+\xi^2-\xi^3,\ |\xi| \ll 1$$

$$\sqrt{1+\xi} \approx 1+\frac{\xi}{2}-\frac{\xi^2}{8}+\frac{\xi^3}{16},\ |\xi| \ll 1$$

$$\arcsin\left(\Xi_0+\xi\right) \approx \arcsin\left(\Xi_0\right)+\frac{\xi}{\sqrt{1-\Xi_0^2}}+\frac{\xi^2\Xi_0}{2\left(1-\Xi_0^2\right)^{3/2}}+\frac{\xi^3\left(2\Xi_0^2+1\right)}{6\left(1-\Xi_0^2\right)^{5/2}},\ |\xi| \ll 1$$

$$\arccos\left(\Xi_0+\xi\right) \approx \arccos\left(\Xi_0\right)-\frac{\xi}{\sqrt{1-\Xi_0^2}}-\frac{\xi^2\Xi_0}{2\left(1-\Xi_0^2\right)^{3/2}}-\frac{\xi^3\left(2\Xi_0^2+1\right)}{6\left(1-\Xi_0^2\right)^{5/2}},\ |\xi| \ll 1$$

$$\operatorname{arctg}\left(\Xi_0+\xi\right) \approx \operatorname{arctg}\left(\Xi_0\right)+\frac{\xi}{\Xi_0^2+1}-\frac{\xi^2\Xi_0}{\left(\Xi_0^2+1\right)^2}+\frac{\xi^3\left(3\Xi_0^2-1\right)}{3\left(\Xi_0^2+1\right)^3},\ |\xi| \ll 1$$

$$\operatorname{arcctg}\left(\Xi_0+\xi\right) \approx \operatorname{arcctg}\left(\Xi_0\right)-\frac{\xi}{\Xi_0^2+1}+\frac{\xi^2\Xi_0}{\left(\Xi_0^2+1\right)^2}+\frac{\xi^3\left(1-3\Xi_0^2\right)}{3\left(\Xi_0^2+1\right)^3},\ |\xi| \ll 1$$

$$\sin\left(\Xi_0+\xi\right) \approx \sin\left(\Xi_0\right)+\xi\cos\left(\Xi_0\right)-\frac{1}{2}\xi^2\sin\left(\Xi_0\right)-\frac{1}{6}\xi^3\cos\left(\Xi_0\right),\ |\xi| \ll 1$$

$$\cos\left(\Xi_0+\xi\right) \approx \cos\left(\Xi_0\right)-\xi\sin\left(\Xi_0\right)-\frac{1}{2}\xi^2\cos\left(\Xi_0\right)+\frac{1}{6}\xi^3\sin\left(\Xi_0\right),\ |\xi| \ll 1$$

$$(1+\xi)^n \approx 1+n\xi+\frac{1}{2}n(n-1)\xi^2+\frac{1}{6}n(n-1)(n-2)\xi^3,\ |\xi| \ll 1$$

P.S.

Neither Hedgehog nor the seagulls were harmed during the filming of this Episode.