



## LPR Cup 2023

9.s04.e01

### Hint 2

**IMPORTANT!** The next task is both a hint and an alternative to the main task. Three important points:

1. You can continue to send the solution to the main problem.
2. At any moment before the final deadline you can start to solve the Alternative problem. If you do so, at the beginning of the solution write: *I am doing the Alternative problem!* In this case a penalty coefficient for the Alternative problem is

$$0,7 \cdot \sum_i \frac{k_i \cdot p_i}{10},$$

where  $p_i$  is a point for the problem item, and  $k_i$  is a penalty coefficient for the corresponding problem's item at the moment of moving to the Alternative problem. In other words, maximal points for the alternative problem equals to the maximal points you can gain at the moment of moving to the alternative one multiplied by 0,7. Also, we remind you that a penalty coefficient can't be less than 0,1. Solutions of the main problems from that moment will not be checked. Be careful!

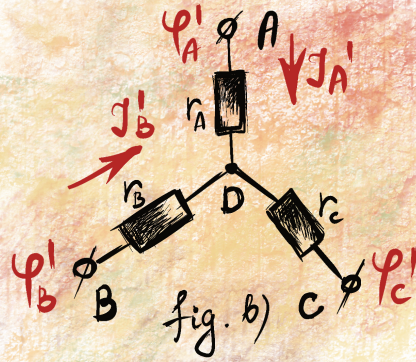
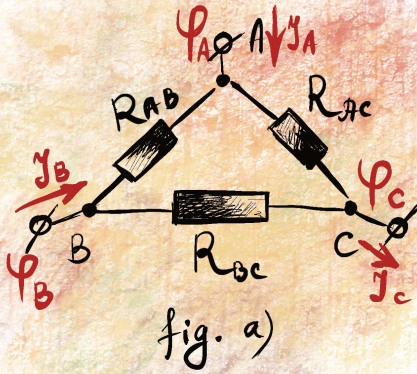
3. The task consists of several items. The penalty multiplier earned **before** is applied to all points. In the future, each item is evaluated as a separate task. If you send a solution without any item, this item's solution is considered as Incorrect. For more information about scoring points for composite tasks, see the rules of the Cup.

### Alternative problem

**1.** A three-terminal (see Fig. a) with three resistors of resistances  $R_{AB}$ ,  $R_{AC}$ ,  $R_{BC}$  is connected with three wires to a circuit. The potentials of the  $A$ ,  $B$ ,  $C$  nodes of the scheme turned out to be equal to  $\varphi_A$ ,  $\varphi_B$  and  $\varphi_C$ , respectively.

1.1 (*0 points*) Express the currents  $I_A$  and  $I_B$  in terms of  $R_{AB}$ ,  $R_{AC}$ ,  $R_{BC}$ ,  $\varphi_A$ ,  $\varphi_B$  and  $\varphi_C$ .

A three-terminal (see Fig. b) with three resistors of resistances  $r_A$ ,  $r_B$ ,  $r_C$  is connected with three wires to the same circuit. The potentials of the  $A$ ,  $B$ ,  $C$  nodes of the scheme turned out to be equal to  $\varphi'_A$ ,  $\varphi'_B$  and  $\varphi'_C$ , respectively.



1.2 (0 points) Express the currents  $I'_A$  and  $I'_B$  in terms of  $r_A, r_B, r_C, \varphi'_A, \varphi'_B$  and  $\varphi'_C$ .

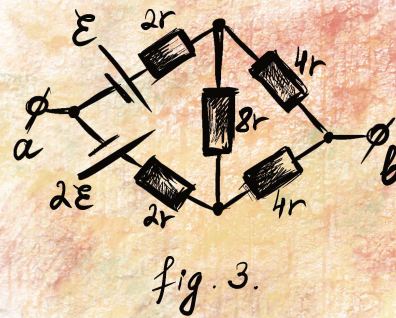
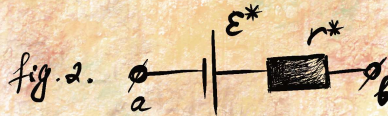
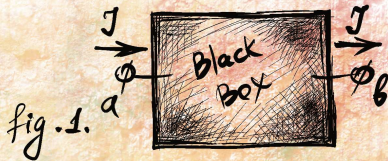
1.3 (0 points) Assuming that  $r_A, r_B, r_C$  are known, determine such  $R_{AB}, R_{AC}, R_{BC}$  that the three-terminal  $\Delta$  (see Fig. a) is equivalent to the three-terminal Y (see Fig. b) when connected to any external circuits with three corresponding pins  $A, B$  and  $C$ . That is, for any  $\varphi_A, \varphi_B, \varphi_C$  and  $\varphi'_A = \varphi_A, \varphi'_B = \varphi_B, \varphi'_C = \varphi_C$ , the following equalities hold:

$$I'_A = I_A, \quad I'_B = I_B.$$

2. Figure 1 shows a two-terminal BB, for which for any current  $I$  the following relation holds:

$$\varphi_B - \varphi_A = A - BI, \tag{1}$$

where  $A > 0, B > 0$  are known constants.

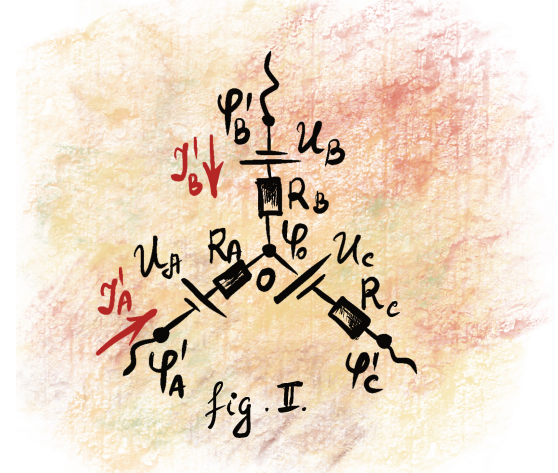
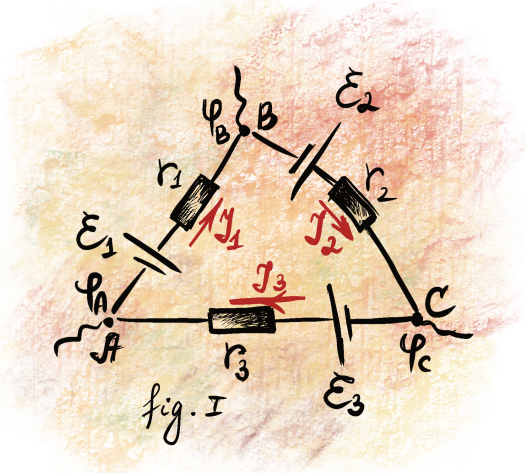


BB can be replaced by an equivalent EMF source (see Fig. 2).

2.1 (1 point) Express the parameters of the equivalent EMF source  $\mathcal{E}^*$  and  $r^*$  in terms of  $A$  and  $B$ .

2.2 (2 points) Show that for the two-terminal in Figure 3, the dependence of  $\varphi_B - \varphi_A$  on  $I$  has the form (1). Determine the parameters  $A$  and  $B$  for this scheme, assuming that  $\mathcal{E}$  and  $r$  are known.

3. Figure I shows an active three-terminal of the  $\Delta$  type connected to an arbitrary circuit. The parameters  $\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, r_1, r_2, r_3$  are considered to be known. It turned out that the currents through the branches of the three-terminal are equal to  $I_1, I_2$  and  $I_3$ .



3.1 (1 point) Express  $\varphi_B - \varphi_A, \varphi_C - \varphi_B$  and  $\varphi_A - \varphi_C$  in terms of the currents  $I_1, I_2$  and  $I_3$  and the parameters of the three-terminal in Fig. I.

Figure II shows an active three-terminal of type Y connected to the same circuit. The parameters  $U_A, U_B, U_C, R_A, R_B, R_C$  are considered to be known. It turned out that the currents through the branches of the three-terminal are equal to  $I'_A, I'_B$ .

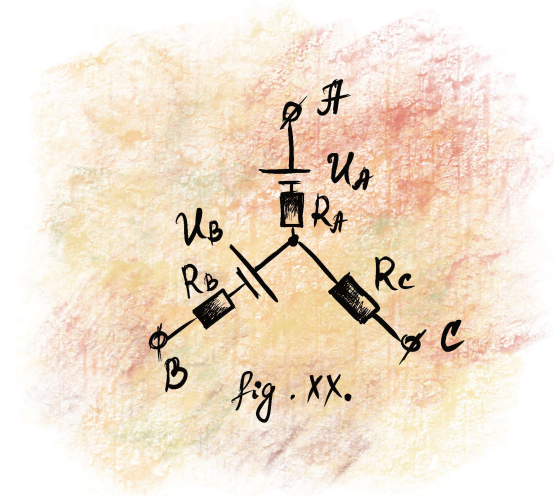
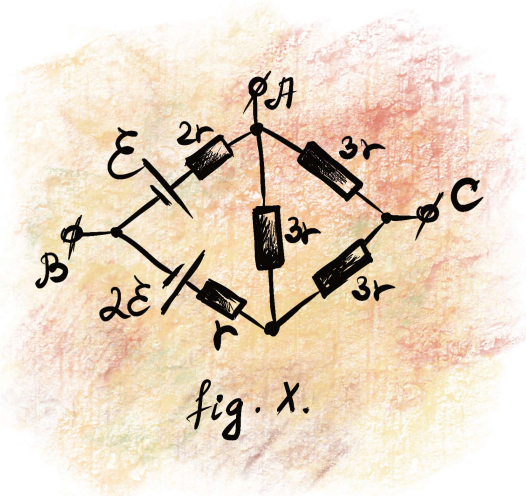
3.2 (1 point) Express  $\varphi_O - \varphi'_A, \varphi_O - \varphi'_B$  and  $\varphi_O - \varphi'_C$  in terms of the currents  $I'_A, I'_B$  and the parameters of the three-terminal II.

3.3 (2 points) Considering  $\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3, r_1, r_2, r_3$  to be known, determine such values of the parameters  $U_A, U_B, U_C, R_A, R_B, R_C$  that the three-terminal II is equivalent to the three-terminal I when connected to any external circuits. That is, the following equalities hold:

$$\varphi'_A = \varphi_A, \quad \varphi'_B = \varphi_B, \quad \varphi'_C = \varphi_C, \quad I'_A = I_1 - I_3, \quad I'_B = I_2 - I_1.$$

3.4 (0 points) What are  $U_A$  and  $U_C$  of the equivalent three-terminal II from the previous paragraph, if we put  $U_B = 0$ ?

4. Figure X shows an active three-terminal X with known parameters  $\mathcal{E}$  and  $r$ .



- 4.1 (1 point) Prove that using Y —  $\Delta$  transformation, discussed in the previous problem, this three-terminal X can be transformed to an equivalent three-terminal of the type Y (see fig. XX).
- 4.2 (2 points) Determine the parameters  $U_A$ ,  $U_B$ ,  $R_A$ ,  $R_B$ ,  $R_C$  of the equivalent three-terminal XX, which is shown on figure XX.