LPR VI Cup

11.s06.e02

There is a theory that the universe and time are infinite,
which means that any event is inevitable,
even the impossible.
Interstate 60

Fermi Paradox

The weather took a turn for the worse.

And then it took a turn for the better.

And Hedgehog was finally able to do his favorite thing — observing the starry sky. His attentive bead-like eyes habitually slid over the constellations, when suddenly he noticed an object whose position and shape did not resemble any celestial body he knew. With trembling paws, Hedgehog quickly recorded its parameters in his notebook, and, running back to the telescope, was unable to find it again.

Time passed, but the mysterious object did not leave our observer's thoughts, and he decided to consult the Archives. In the dusty tomes, Hedgehog found only one similar mention and began to think even more about the complicated fate of the celestial body.

Relying on his own notes and the records from the Archives, Hedgehog managed to determine that the angle between the directions of observation of this object is α, and using special methods of motion analysis, he calculated that the angle between the directions of the velocities of the same object at the moments of observation is β.

1. *(1 point)* What type of trajectory (parabola, hyperbola, ellipse) is possible under the condition that $\alpha = \beta$? Provide one example satisfying the condition for each possible trajectory, and prove that other types of trajectories are impossible.
2. *(3 points)* Assuming both values α and β are random and uniformly distributed in the range $[0; \pi]$, determine the probability that the object is **definitely NOT** of alien origin, i.e., the final values of α and β correspond **only** to a closed type of trajectory.

Using even more advanced methods of analysis, based on the signal obtained at the moments of observation, Hedgehog managed to find out that the ratio of the magnitudes of the velocities at the moments of observation was k. Determine the ratio of the distances to the object at the moments of observation, if:

3. *(1 point)* $\alpha = \pi/3$, $\beta = \pi/3$, $k = 1$.
4. *(1 point)* $\alpha = \pi/3$, $\beta = \pi/6$, $k = 1$.
5. *(4 points)* $\alpha = \pi/3$, $\beta = \pi/4$, $k = \frac{\sin(2\pi/3)}{sin(\pi/12)+sin(\pi/4)}$.

In all parts, assume that the observations were made practically from the center of the Sun (the distance to the object at any moment is much greater than the distance between the

Earth and the Sun), and that the object moves freely in the gravitational field of our star (it does not experience gravitational interaction with other bodies and does not change its momentum due to reactive forces).

Zero Hint — 04.05.2025 15:00 (Moscow time)
First Hint — 05.05.2025 20:00 (Moscow time)
Second Hint — 07.05.2025 12:00 (Moscow time)

Final of the first round — 09.05.2025 20:00 (Moscow time)