LPR Cup

11.s05.e05

Hint 2

IMPORTANT! The next task is both a hint and an alternative to the main task. Three important points:

1. You can continue to send the solution to the main problem.
2. At any moment before the final deadline you can start to solve the Alternative problem. If you do so, at the beginning of the solution write: *I am doing the Alternative problem!* In this case a penalty coefficient for the Alternative problem is

$$0{,}7 \cdot \sum_i \frac{k_i \cdot p_i}{10},$$

where p_i is a point for the problem item, and k_i is a penalty coefficient for the corresponding problem's item at the moment of moving to the Alternative problem. In other words, maximal points for the alternative problem equals to the maximal points you can gain at the moment of moving to the alternative one multiplied by 0,7. Also, we remind you that a penalty coefficient can't be less than 0,1. Solutions of the main problems from that moment will not be checked. Be careful!

3. The task consists of several items. The penalty multiplier earned **before** is applied to all points. In the future, each item is evaluated as a separate task. If you send a solution without any item, this item's solution is considered as Incorrect. For more information about scoring points for composite tasks, see the rules of the Cup. **Since switching to an alternative selection, there is no opportunity to return to solving the main task.** Also, after switching to an alternative task **the points for the main task are reset**.

Introduction

There were drafts next to the folder, from which an attentive Observer could find out how Hans came to certain results of his main work.

Alternative task

Part 1. Why do nuclei reactions matter?

The correct dice was rolled 3 times and the sum of the results turned out to be 6.

1. *(0 points)* What are the possible sequences of dice roll results?
2. *(0 points)* What is the most likely sequence?
3. *(1 point)* Which result set is the most likely?

Part 2. Thermal conductivity

This part does not help in solving the main problem. It is designed to deepen your understanding of the operation of thermonuclear reactors.
Let's study the issues related to thermonuclear fusion. It is known that in the Universe thermonuclear reactions occur in the bowels of stars at high (on the order of 10^7 K) temperatures. There, the plasma is held by enormous gravitational forces [1]. Even for relatively low densities of plasma matter, the star remains stable. When creating an installation that performs thermonuclear fusion in laboratory conditions, a problem arises. Firstly, it is very difficult to keep plasma in a limited volume. Secondly, plasma particles move towards the walls of the reactor, thereby creating heat flows of enormous power.

Let's estimate the thermal power coming to the walls of the reactor. Let's define the heat flow j as the amount of heat passing through a unit area of some site per unit time. For simplicity, we will consider a one-dimensional situation. Let $\varepsilon(x) = c_V T(x)$ — the energy of a molecule located at the coordinate x, where c_V — the heat capacity per molecule. $N^{(\uparrow)}$ and $N^{(\downarrow)}$ — the number of molecules passing through the plane located at the x coordinate during the free run. It is clear that if the gas as a whole does not move anywhere, then $N^{(\uparrow)} = N^{(\downarrow)}$.

1. *(1 point)* Print the formula for the heat flow. Use the following calculation plan:
   (a) Write down the energy that passes through the plane located at the x coordinate.
   (b) Simplify the expression using the equalities $N^{(\uparrow)} = N^{(\downarrow)}$ and $\varepsilon(x) = c_V T(x)$.
   (c) Write down the final expression for the heat flow in the form $j = \alpha \dfrac{dT}{dx}$, where α is called the coefficient of thermal conductivity. Express α in terms of free path λ, particle velocity v, plasma density ρ and specific heat capacity of plasma $c_V^{(m)}$ with constant volume. Specific heat capacity is the heat capacity per unit mass.
2. *(1 point)* Estimate the numerical value of the heat flow at the thickness of the transition layer from the reactor core to the environment in 1 km. The characteristic temperature of the core is $T = 10^8$ K. There is the following hypothetical solution to the problem of retaining plasma particles: it is proposed to place it in a strong magnetic field. Then the charged particles will describe helical trajectories around the magnetic field lines, and the space of the plasma particles will be limited.
3. *(1 point)* Find the characteristic magnitude of the magnetic field induction necessary to hold the particles — i.e., so that the particles, due to their thermal motion, do not significantly move perpendicular to the lines of magnetic induction. Consider the known

[1] The masses of the stars are very large. For example, the mass of the Sun is approximately 10^{30} kg.

masses of particles and their charges (particles — electrons), the temperatures of the components ($T = 10^8$ K), as well as the concentration ($n = 10^{15}$ cm^{-3}).

Part 3. Plasma

1. *(0 points)* The space is filled with an ideal gas with a particle radius of R and a concentration of n. Estimate the average particle path length. Find the numerical value for $R = 10^{-10}$ m and $n = 10^{25}$ m^{-3}. Consider that molecules do not interact with each other at a distance, but when approaching at a distance less than the sum of the radii, a collision occurs.
2. *(0.25 points)* A fast electron flies into the region where neutral atoms with a concentration of n are located. Each time it hits an atom, it loses $\alpha = 0.1\%$ of its energy. Consider the effective area for calculating impacts to be equal to S. Estimate the distance at which the electron will lose half of its energy. Find the numerical value for $n = 10^{25}$ m^{-3} and $S = 4 \cdot 10^{-20}$ m^2.
3. *(0.25 points)* Find the electric field strength for a uniformly charged infinite layer with a thickness of h. The bulk charge density is ρ. Plot the dependence of the electric field strength on the distance to the central section of the layer.
4. *(0.25 points)* A narrow through channel is made across a uniformly charged infinite layer with a thickness of $2d$ and with a charge density of ρ. An electron flies into the channel at a speed of v_0. Find the velocity of the electron in the middle and at the exit of the channel.
5. *(0.25 points)* An electric dipole of mass m and with a dipole moment p flies into the channel at a speed of v_0. The speed of the dipole increases. Find the velocity of the dipole in the middle of the channel. Find the velocity of the dipole at the outlet of the channel.

Part 4. Déjà vu

This section challenges you to derive a criterion that, when satisfied, ensures the operation of a fusion power plant at zero net power output. Consider a finite volume containing a plasma composed of deuterium and tritium nuclei, along with electrons resulting from their ionization. All plasma components behave as an ideal gas. The number of deuterium-tritium fusion reactions

$$d + t \longrightarrow {}^4_2\text{He} + n + 17.6\text{MeV}$$

per unit volume per unit time is given by:

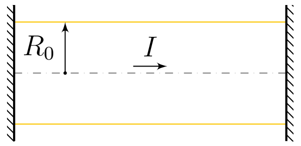
$$A \cdot n^2 \cdot T^{1/2},$$

where n is the known concentration of each type of nuclei within the reactor, T is the known temperature of all plasma components, and A is a known constant.

Assume that all reaction products reach the reactor walls without interacting with the plasma, transferring all their kinetic energy as heat. The walls, in turn, convert this heat into electricity, which is then used to heat/maintain the plasma temperature. This feedback system operates with efficiency η. It is known that if the feedback mechanism and nuclear reactions are "stopped the plasma will cool down with a characteristic cooling time τ, which is determined by the reactor's design features and is one of its key parameters. Here, the characteristic cooling time is the time it would take for the plasma to reach zero temperature if it were to cool at a constant rate equal to the initial rate.

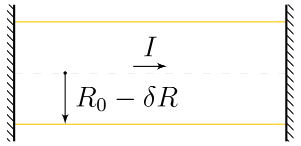
1. *(1 point)* Under what condition on the value of $n\tau$ is it possible for the reactor to operate in such a way that the described system can operate indefinitely and "self-sustain"due to the feedback mechanism?

Part 5. Hcnip-Z



Consider a conductive shell in the shape of a cylinder. Within the framework of this part, consider that the shell can deform (compress and decompress) without losing its shape and conductive properties. Ignore the elastic force of the conductive shell. A current I flows along the surface of the conductive shell. Inside the shell there is a monatomic gas at a pressure of p_0.

1. *(0.5 points)* What current should flow through the conductive shell so that it is kept in equilibrium by its own magnetic field?



Consider a situation in which a uniform compression of the conductive shell occurred along the entire length (see figure).

2. *(1 point)* Analyze the stability of the equilibrium position from the previous point. In the case of a stable equilibrium position, find the oscillation period. Consider the process in isothermal and adiabatic approximations.

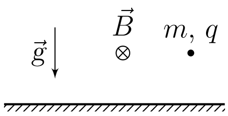
The initial radius of the cylinder is R_0, the mass of the shell per unit length is λ, the initial pressure inside the shell is p_0.

Part 6. Srorrim

Consider that during the transition between systems, the electromagnetic field is transformed as follows:

$$\begin{cases} \vec{E}_1 = \vec{E}_0 - \left[\vec{v} \times \vec{B}_0\right], \\ \vec{B}_1 = \vec{B}_0 + \frac{1}{c^2}\left[\vec{v} \times \vec{E}_0\right]. \end{cases}$$

Here $\vec{v}$ — the velocity of the reference frame is 1 in the reference frame 0, $v \ll c$.



A ball of mass m with a positive charge q begins to move in gravitational and magnetic fields. Consider the fields homogeneous. The magnetic field induction is equal to B and is directed parallel to the horizon (see figure).

1. *(2 points)* How far will the ball move in the laboratory frame of reference after a sufficiently long time T.

2. *(0 points)* In which frame of reference will the electric field compensate for the force of gravity?

3. *(0 points)* How will the ball move in this frame of reference?

Consider that there is no impact with the Earth's surface. $qcB \gg mg$.

Part 7. Kamakot

Drift in an inhomogeneous magnetic field

1. *(0.5 points)* Space is divided into two regions by a plane through which charged particles can freely pass. In one region of space, the magnetic field induction is equal to B_1, in the other — B_2. The fields are homogeneous and parallel to each other. Perpendicular to the interface plane, a particle is released at a velocity of $v_\perp$ to the side with an induction of B_1. Describe the further movement of the particle. Determine the drift velocity of the particle.
2. *(0 points)* Will the scorsole from the previous paragraph be the same in modulus for particles with anti-cold charges? In the direction?
3. *(0 points)* Draw conclusions based on the previous paragraph if there are many positively charged (ions) and negatively charged (electrons) particles in space. Will there be any force keeping these charges from drifting?

Magnetic fields

4. *(0 points)* Find the induction of a magnetic field at a distance a from an infinite straight wire if a current I flows through the wire.
5. *(0 points)* Find the induction of the magnetic field at a distance a from the annular wire, if a current flows through the wire I, and the radius of the conducting ring is $R \gg a$. Compare with the previous paragraph.
6. *(0 points)* Find the induction of the magnetic field inside an infinitely long solenoid, if a current I flows through the winding of this solenoid, the winding density is n.
7. *(0 points)* Find the induction of the magnetic field inside the toroidal coil if the small radius of the torus is a, large — R, winding density n, current I. Consider that $R \gg isa$. Compare with the previous paragraph.

Consider that there are no other sources of the magnetic field in the problem.

Helix line

8. *(0 points)* The center of rotation of a certain point moves uniformly along the OZ axis at a speed of v_z. The point rotates in the XY plane around this center. Radius of rotation — a, speed —v_φ. How should the speeds of v_z and v_φ, so that the particle returns to the initial coordinates x, y (but not z), passing along the axis OZ a distance equal to $2\pi Rq$?
9. *(0 points)* Solve the previous problem on the torus for which $a \ll R$. Schematically draw the trajectory of the particle.

Consider the distances $2\pi Rq$ and a known.