LPR Cup

11.s05.e04

Hint 2

IMPORTANT! The next task is both a hint and an alternative to the main task. Three important points:

1. You can continue to send the solution to the main problem.
2. At any moment before the final deadline you can start to solve the Alternative problem. If you do so, at the beginning of the solution write: *I am doing the Alternative problem!* In this case a penalty coefficient for the Alternative problem is

$$0{,}7 \cdot \sum_i \frac{k_i \cdot p_i}{10},$$

where p_i is a point for the problem item, and k_i is a penalty coefficient for the corresponding problem's item at the moment of moving to the Alternative problem. In other words, maximal points for the alternative problem equals to the maximal points you can gain at the moment of moving to the alternative one multiplied by 0,7. Also, we remind you that a penalty coefficient can't be less than 0,1. Solutions of the main problems from that moment will not be checked. Be careful!

3. The task consists of several items. The penalty multiplier earned **before** is applied to all points. In the future, each item is evaluated as a separate task. If you send a solution without any item, this item's solution is considered as Incorrect. For more information about scoring points for composite tasks, see the rules of the Cup. **Since switching to an alternative selection, there is no opportunity to return to solving the main task.** Also, after switching to an alternative task **the points for the main task are reset**.

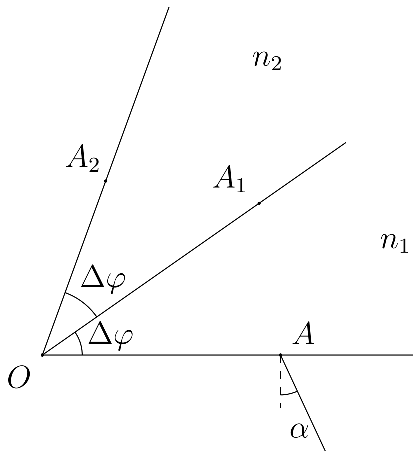
Alternative task

Special case

The acoustic refractive index depends only on the angle φ. This dependence has the form

$$\begin{cases} n(\varphi) = n_1, & \text{when } 0 < \varphi \le \Delta\varphi, \\ n(\varphi) = n_2, & \text{when } \Delta\varphi < \varphi \le 2\Delta\varphi, \\ n(\varphi) = 1, & \text{otherwise.} \end{cases}$$

The ray crosses the line OA at the point A with an angle of incidence α (see figure).



1. *(2 points)* Find the angle at which the ray will exit the area with $n(\varphi) = n_2$ (i.e., the angle between the perpendicular to OA_2 and the part of the ray in the area $\varphi > 2\Delta\varphi$).
2. *(2 points)* Calculate $n(2\Delta\varphi)\sin(\alpha_2)$ and $n(\Delta\varphi)\sin(\alpha_1)$, where α_1 and α_2 are the angles of incidence at OA_1 and OA_2 accordingly.
3. *(2 points)* Find the time it took for the ray to exit the area $0 \le \varphi \le 2\Delta\varphi$. The length of the segment OA is equal to R.

Assume that α and $\Delta\varphi$ are small angles.

General case

4 *(3 points)* Consider the region $\varphi_1 \le \varphi \le \varphi_2$, in which the refractive index is $n(\varphi)$. The angles of incidence of the beam on the straight lines $\varphi = \varphi_1$ and $\varphi = \varphi_2$ are equal to α_1 and α_2, respectively (angles of incidence are measured as shown on the figure above). The ray intersects these lines at points located at distances R_1 and R_2 from the origin. Prove that

$$s = n(\varphi_1)R_1 \sin\alpha_1 - n(\varphi_2)R_2 \sin\alpha_2,$$

where s is the optical path that the ray has traveled in this area. n depends on φ continuously.

Cord

5 *(1 point)* There is a homogeneous explosive cord. The velocity of propagation of the explosion reaction along the cord is v, and the velocity of propagation of the blast wave

through the air is c. Find the shape of the line $r(\varphi)$ along which you need to position the cord so that the wave from all points of the cord comes to a point with polar coordinates $r = 0$ at the same time. Assume that $c < v$.