LPR Cup

11.s05.e04

Move like a beam of light: fly like lightning, strike like thunder, whirl in circles around a stable center.
Morihei Ueshiba

Geometric acoustics

"Fog" - thought Hedgehog and felt like home.

He liked the island immediately, and he decided to separate from the group for a short time and wander in the synoptic conditions that were natural for him. He walked further and further, going deep into the island, where the fog became thicker and thicker, and the cries of the seagulls became quieter and quieter. And at some point, it is not known exactly which one, because Hedgehog, as always, lost track of time, he almost stumbled upon a weakly trodden path. Hedgehog was delighted with it, because besides time, he had already lost his way as well. And even the compass, presented by one of the gentlemen, could not help him, because its needle rather phlegmatically walked in a circle, as if in childhood it dreamed of becoming a second hand, and not at all an arrow of a compass.

Hedgehog walked along the winding path and imagined that it was not it which twisting so much in bizarre zigzags, but the world around. He walked for a long time, for a very long time, but at some point he noticed that the fog began to dissipate, and the compass needle began to calm down. It no longer spun as if it were the hand of a clock, but all the time showed the same direction - along a winding path. Hedgehog continued on and soon, in a faint haze of fog, he saw the dome of the observatory. Never in his life had he been as happy as he was at that moment. His paws began to jog, and the observatory, which did not loop as the world around it approached faster and faster.

There was no one inside the observatory, but it was very cozy. The first thing Hedgehog did was looking through the telescope and saw that it was tuned to some distant planet, the surface of which was decorated with craters of various sizes.

Tearing himself away from the fascinating view with difficulty, the Hedgehog looked around and only now noticed a large laboratory table with a large number of interesting devices in the center of the room. He climbed onto a high stool with difficulty and began to study them carefully. The Hedgehog's attention was attracted by two polished discs made of a very strange cold material, on the surfaces of which acoustic sensors and sources were attached. Next to the first disc was the tag «Parabola», and with the second «Iron IIIB-Om». Not far from them lay an open laboratory journal.

From these studies, the Hedgehog learned that the propagation of sound in an environment with an acoustic refractive index n is similar to the propagation of light and Snell's law is exactly the same for it. It was accepted in the entire laboratory journal that at $n = 1$ the propagation velocity of the acoustic beam is c, and all positions of points in the plane of the disk are set in coordinates x/l, y/l. The distance l is known.

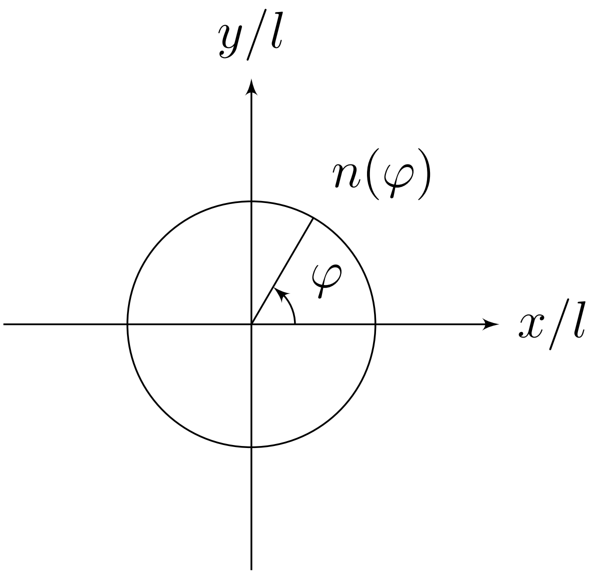
Part 1. Parabola

The acoustic refractive index of the disk depends only on the angle φ. At the point $A(0;1)$, the refractive index is $n(\varphi) = 1$.

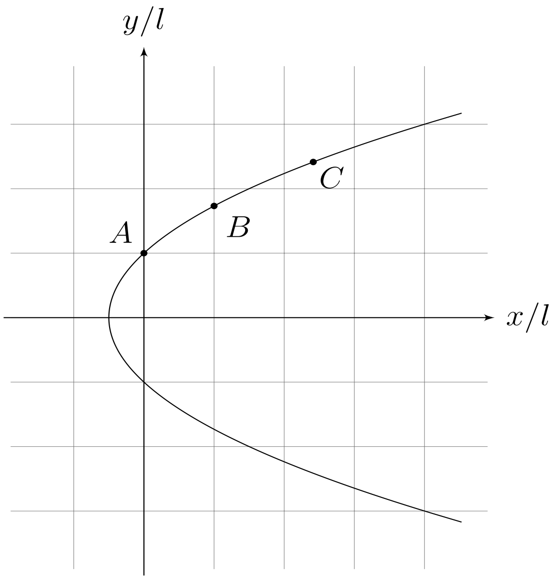
1. *(0 points)* Find the angle φ for the point A.

The trajectory of an acoustic ray has a form of parabola Π (see Fig. 1b), passing through points $A(0;1)$, $B\left(1;\sqrt{3}\right)$, $C\left(1+\sqrt{2};1+\sqrt{2}\right)$

2. *(2 points)* Find the time it takes for the ray to reach point $B\left(1,\ \sqrt{3}\right)$ if it is launched from point A.
3. *(2 points)* Find the expression for the refractive index $n(\varphi)$.



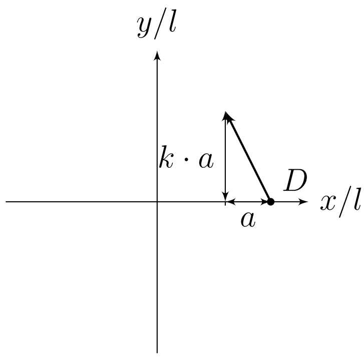
(a) Polar coordinates



(b) Parabola Π

Part 2. Iron IIIB-Om

From the laboratory journal, the Hedgehog learned that an acoustic beam is emitted from the point D(4,0) at some angle (see Fig. on the right) The refractive index of the disc material along the trajectory of the beam varies according to the law $n = e^{-\varphi \cdot k}$, $k > 0$.



1. *(2 points)* Find the equation of the ray trajectory.
2. *(2 points)* After what time will the ray cross the OY axis for the first time?
3. *(2 points)* Under what condition will the angular velocity of the beam be minimal after a sufficiently long time?

Consider that the angle changes continuously along the trajectory.

The following formula may be useful to you:

$$\frac{d}{dx}\left(\sin\frac{x}{2}\right) = \frac{1}{2}\cos\frac{x}{2}$$

First hint — 20.05.2024 20:00 (Moscow time)
Second hint — 22.05.2024 12:00 (Moscow time)

Final of the forth round — 24.05.2024 20:00 (Moscow time)