LPR Cup

11.s05.e03

Hint 2

IMPORTANT! The next task is both a hint and an alternative to the main task. Three important points:

1. You can continue to send the solution to the main problem.
2. At any moment before the final deadline you can start to solve the Alternative problem. If you do so, at the beginning of the solution write: *I am doing the Alternative problem!* In this case a penalty coefficient for the Alternative problem is

$$0{,}7 \cdot \sum_i \frac{k_i \cdot p_i}{10},$$

where p_i is a point for the problem item, and k_i is a penalty coefficient for the corresponding problem's item at the moment of moving to the Alternative problem. In other words, maximal points for the alternative problem equals to the maximal points you can gain at the moment of moving to the alternative one multiplied by 0,7. Also, we remind you that a penalty coefficient can't be less than 0,1. Solutions of the main problems from that moment will not be checked. Be careful!

3. The task consists of several items. The penalty multiplier earned **before** is applied to all points. In the future, each item is evaluated as a separate task. If you send a solution without any item, this item's solution is considered as Incorrect. For more information about scoring points for composite tasks, see the rules of the Cup. **Since switching to an alternative selection, there is no opportunity to return to solving the main task.** Also, after switching to an alternative task **the points for the main task are reset**.

Alternative task

The grinding in the corridor begins to subside little by little, but you prudently do not rush to leave the place that was once used as an office. Having carefully examined the videographer, clicking all the toggle switches and examining the map, you wait for the noise in the corridor to subside to zero.

Mechanically opening the lid of the breguet, you see that the watch is standing still. There is a feeling that you need to be surprised, but this place has taught you not to do it. You phlegmatically decide that since time stands still, it must either be spent usefully or killed. In the absence of anything better, you begin to study the cabinet in more detail and soon notice that the breguet's hands move with (or maybe after?) you. After making a few confusing movements around the office and closely following the second hand of the clock, you realize that it does not move by chance, but points to the same place in the office.

Inside you, everything becomes wobbly from recent memories of what was in the corridor, but you still go to the part of the wall of the office that the clock hands point to. Having carefully examined it, you notice that one of the wall tiles protrudes slightly more than the others. With a trembling hand, you press it and hear an old mechanism triggered, and an ingenious system of levers and counterweights opens a small hiding place.

You turn up the light of the gas-discharge lamp and see an old notebook lying in it. Carefully leafing through it, you realize that these are the drafts of the one who worked here on all these mechanisms that fill this damn tunnel.

Notes from the notebook. Part 1

Three-dimensional electrostatics

Consider an arbitrary closed surface S, inside which there is a total charge Q. By Gauss's theorem,

$$\Phi = \frac{Q}{\varepsilon_0},$$

where Φ is the electric field $\vec{E}$ flux through the surface S:

$$\Phi = \int\limits_S E_\perp dS.$$

Here $E_\perp$ is the projection of the vector $\vec{E}$ onto the outward normal to the surface S.

Using Gauss's theorem, we can find, for example, the field of a point charge Q. Let's consider a sphere of radius r centered at the point where the charge is located. Due to symmetry, we infer that the field intensity of the charge is uniform in magnitude at all points on the sphere and perpendicular to it. Then

$$\frac{Q}{\varepsilon_0} = \int\limits_S E(r)\, dS = E(r) \int\limits_S dS = E(r) 4\pi r^2.$$

From this, we obtain

$$E(r) = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}.$$

1. *(1 point)* Find the intensity of the electric field of a long thin straight wire, with a charge density of $\varkappa$ per unit length.
2. *(1 point)* Find the electric potential of a long thin straight wire, with a charge density of $\varkappa$ per unit length.

Two-dimensional electrostatics

3. *(1 point)* Formulate the analogue of Gauss's theorem for the two-dimensional case.
4. *(1 point)* Find the field of a point charge in the case of two-dimensional electrostatics.
5. *(1 point)* Find the potential of a point charge in the case of two-dimensional electrostatics.

Disk

In Cartesian coordinates, in the two-dimensional case, the gradient of the function $f(x,y)$ at the point (x, y) is a vector that is equal to

$$\mathrm{grad} f(x,y) = \frac{\partial f}{\partial x}\vec{e}_x + \frac{\partial f}{\partial y}\vec{e}_y,$$

where $\vec{e}_x$ and $\vec{e}_y$ are unit vectors directed along the OX and OY axes.

6. *(0 points)* Compute the gradients of functions $f(x,y) = axy^2$ and $g(x,y) = b(x^2 + y^2)$, where a и b are known constants.

Gradient of a function $f(x,y) = f(\rho)$, that depends only on $\rho = \sqrt{x^2 + y^2}$ is easier to compute in polar coordinates. It is equal to

$$\mathrm{grad}\, f(\rho) = \frac{df}{d\rho}\vec{e}_\rho,$$

where $\vec{e}_\rho$ — is a unit vector that is parallel to the radius vector at every point.

7. *(2 points)* The disk is placed again in the center of the film, as in the first part of the main problem. Consider a circle γ with a radius r, whose center coincides with the center of the film. Find the flux of the field $\mathrm{grad}\, z(x,y)$ through γ.

Notes from the notebook. Part 2

In the hiding place you find an old and battered film and two more notebooks, each of which looks older than this tunnel.

Notebook 1

Notebook 2

Power up the videographer and watch the recording

A long thin wire is placed parallel to the axis of a long metallic cylinder with radius R at a distance $r < R$ from its axis. Far from its edges, the charge density is $\varkappa$ per unit length on the wire and $-\varkappa$ on the cylinder.

8. *(2 points)* Find the image of the wire in the cylinder.
9. *(1 point)* Find the interaction force between the conductors per unit length.