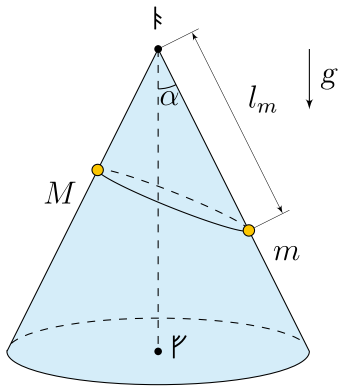
LPR Cup

11.s05.e01

The journey is what brings us happiness not the destination
Dan Millman, «Way of the Peaceful Warrior»

Pilgrim Ant

Part 1. Statics

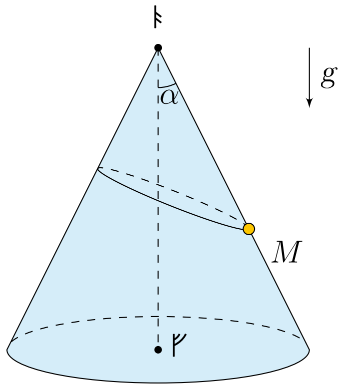


One sunny day Haymaker-Spider and Traveler-Ant found a bag made from a glossy Vogue magazine. It had a form of a right circular cone with an determined angle of α and was located so that its axis ᚨᚴ was aligned vertically. The free fall acceleration is g.

Glossy magazines are smooth, and in order to stay on the bag, Spider wove two weightless inextensible identical threads of length l and with their help hung together with Ant on the cone as shown in the figure. It is well known that the web of Haymaker-Spider is not sticky, so there is no friction between it and the bag.

Considering the masses of Ant and Spider equal to m and M, respectively, determine:

1. *(0,5 points)* The tension force of the threads T_0 in the equilibrium position. In this item the masses of Spider and Ant are the same, and the angle $\alpha = \pi/6$;
2. *(1 point)* At what distance l_m from the vertex ᚨ will Ant be located in a stable equilibrium position if the insect masses differ;
3. *(1 point)* The tension forces of the threads T in the equilibrium position;
4. *(0.5 points)* At what angles α is such an equilibrium possible.
5. *(1 point)* Ant was shifted down along the generatrix for a short distance and released; determine the period of the resulting oscillations; at this point, consider that $m = 2M$.



When Ant ran home, Spider wove a massive homogeneous inextensible thread of unknown length and constant thickness and hung with it on a bag (see Fig). The ends of the thread are attached to the insect. It is known that the minimum and maximum distances from various points of the thread to the vertex ᚨare $l_{\min} = l_0$ and $l_{\max} = 2l_0$, respectively, where l_0 is an unknown quantity. The linear density of the thread is $\lambda = M/l_0$. Determine:

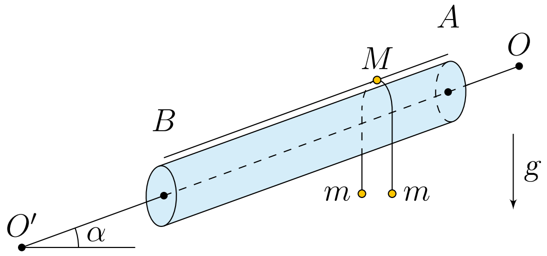
6. *(1.5 points)* The maximum tension force of the thread $T_{\max}$

In the wind, the bag, together with Spider, spun around the vertical axis ᚨᚴ, so that this mechanical system rotates with a certain constant angular velocity ω. It is known that the distance from any point of the thread to the tip of the cone ᚨ does not change during movement, and the minimum and maximum distances are $l'_{\min} = al_0$ and $l'_{\max} = bl_0$, respectively, and $a \neq b$, and $\omega^2 = 2fg/l_0$, where a, b, f are known constants. The thread and Spider do not come off the surface of the bag. Determine:

7. *(0.5 points)* How much do the maximum and minimum thread tension forces differ?

Part 3. Dynamics

A small bead of mass M can slide without friction along a long thin rod AB, which is infinitely close to the uppermost forming a long smooth cylindrical straw of radius R. The axis of the straw OO' is inclined to the horizon at an angle α. Small identical spiders with masses of m ($m \gg M$) were attached to the bead with two weightless threads from a $5R$-long webs. Acceleration of gravity g. Consider the threads elastic and inextensible.



The bead is held in a certain position, the system is in equilibrium. Determine:

1. *(0.5 points)* What is the minimum external force F_0 to be applied to the bead for this.

The bead is released. Determine:

2. *(0.5 points)* Acceleration of spiders a_g immediately after the bead is released;
3. *(1 point)* At what time τ the line connecting the spiders moves away for the first time to the maximum distance l_m from the OO';
4. *(0.5 points)* This distance;
5. *(1 point)* The distance from the axis l_2 will the spiders be after the time of 2τ after the bead is released;
6. *(0.5 points)* The speed of spiders v_2 at this moment.

First hint — 29.04.2024 20:00 (Moscow time)
Second hint — 01.05.2024 12:00 (Moscow time)

Final of the first round — 03.05.2024 20:00 (Moscow time)