

LPR Cup 11.s03.e04



## Hint 2

**IMPORTANT!** The next task is both a hint and an alternative to the main task. Three important points:

- 1. You can continue to send the solution to the main problem.
- 2. At any moment before the final deadline you can start to solve the Alternative problem. If you do so, at the beginning of the solution write: *I am doing the Alternative problem!* In this case a penalty coefficient for the Alternative problem is

$$0,7 \cdot \sum_{i} \frac{k_i \cdot p_i}{10},$$

where  $p_i$  is a point for the problem item, and  $k_i$  is a penalty coefficient for the corresponding problem's item at the moment of moving to the Alternative problem. In other words, maximal points for the alternative problem equals to the maximal points you can gain at the moment of moving to the alternative one multiplied by 0,7. Also, we remind you that a penalty coefficient can't be less than 0,1. Solutions of the main problems from that moment will not be checked. Be careful!

3. The task consists of several items. The penalty multiplier earned **before** is applied to all points. In the future, each item is evaluated as a separate task. If you send a solution without any item, this item's solution is considered as Incorrect. For more information about scoring points for composite tasks, see the rules of the Cup.

## Alternative problem

- 1. (0 points) Gravitational acceleration on the surface of the planet from an incompressible fluid is  $a = 9.8 \frac{\text{m}}{\text{s}^2}$ . Find the pressure at the center of the planet.
- 2. (0 points) Let's consider a narrow layer of thickness  $\Delta x$  and area S such that on one side it is heated to temperature  $T_1$  and on the other side to temperature  $T_2$ . Power equal to the amount of heat that was transmitted over a short time interval  $\Delta t$  from one surface to another is:

$$P = \frac{\varkappa}{\Delta x} S(T_2 - T_1),$$

where  $\varkappa$  is the heat transfer coefficient.

A party popper made of a material with variable thermal conductivity has the shape of a cylinder of radius  $R_2$  with a cylinder of radius  $R_1$  cut out of it. The thermal conductivity depends on the radius as follows:

$$\varkappa(R) = \varkappa_1 \cdot \frac{R^2}{R_1^2}$$

Cylinders  $R_1$  and  $R_2$  are maintained at temperatures  $T_1$  and  $T_2$  respectively. Find the distribution T(R) and the total power transferred from the inner cylinder to the outer one.



- 3. (0 points) Point K is a point on the PV-diagram that describes the state of a constant amount of a monatomic ideal gas. The slope angle of the isotherm at this point to the V-axis is equal to a. What is the angle of inclination of the adiabat at the same point to the V-axis. Explain the answer.
- 4. The Hedgehog walked around the physics lab and found an interesting installation! The installation was a horizontal heat-insulated cylindrical tube and inside there were two metal pistons connected to a constant voltage source. Between the pistons is an ideal (like a white horse!) monatomic gas. The source voltage is U, and the distance between the pistons is d. The Hedgehog denoted  $P_{\rm g}$  as a pressure on the piston caused by the ideal gas and  $P_{\rm el}$  as a pressure on the pistons caused by the electric interaction with each other. It is known that at some point in time  $d = d_1$ ,  $P_{\rm g} = P_{\rm g1}$ . Ignore the friction of the pistons against the walls of the vessel, atmospheric pressure, and the heat in the lead wires.
  - (a) (1 point) Help the Hedgehog to find the distance between the pistons  $d_0$  in the equilibrium position.
  - (b) (1 point) The Hedgehog wants to plot the dependence  $P_{\rm g}(d)$  and  $P_{\rm el}(d)$ , but he has paws. Do it for him.
  - (c) (1 point) Help the Hedgehog to understand, whether there will be the equilibrium position found in part 1 be stable?



- 5. Walking on an elephant, the Hedgehog discovered the Flat World. The Flat World is a thin infinite homogeneous plate with a surface density  $\sigma_0$ . The atmosphere of the Flat World consists of two symmetrical layers of monatomic gas on either side of the plate.
  - (a) (2 points) Find the dependence of the gravitational acceleration in the atmosphere of the Flat World on the distance x to the Flat World. Consider the function

$$\sigma(a) = \int_0^a \rho(x) dx$$
 as known, where  $\rho(x)$  is the gas density.

- (b) (1 point) Express the equilibrium pressure gradient  $\frac{dP}{dx}$  in terms of  $\rho(x)$  and  $\sigma(x)$ .
- (c) (2 points) Express the pressure gradient in terms of mass  $\frac{dP}{d\sigma}$ .
- (d) (1 point) Find the gas pressure on the surface of the Flat World, if its total surface density  $\sigma(x_{\text{max}})$  is known.
- (e) (1 point) Specify, if the Flat World atmosphere is resistant to collapse while breathing?

