



*Humans are constantly coming up with hypotheses,
even when they're being cautious,
and even though they're quite unaware of it.*

Stanislav Lem. Solaris

Spiky hair and sharp eye

Every spring Hedgehog conducts a series of astronomical observations. This year looking through a telescope twice, he was able to observe a star around which there were two interesting planets moving.

The porous planet

(5 points) The first planet is made of a porous material filled with polyatomic nonlinear ideal gas. There are no reasons for Hedgehog to believe that the density of the porosity material is a lot more than gas density. Hedgehog wonders if the planet's gas component is «breath»-stable, i. e. can its size weakly spontaneously homothetically and adiabatically oscillate. Help Hedgehog solve this puzzle.

Note. In the following parts of the problem, we assume that if the planet is «breath»-stable, then it is generally stable and can exist.

The hot planet

(2 points) Analysis of the second planet shows that it fully consists of ideal diatomic gas without the solid or liquid core. The heat capacity of the planet gas at a constant volume depends on the temperature according to the following law:

$$C_V = \frac{5}{2}R + \left(\frac{A}{T}\right)^2 \frac{1}{\text{sh}^2(A/T)}R,$$

Here $A = 5000 \text{ K}$. Determine the maximum possible temperature of the core of the second planet, considering that it is less than a critical one for a given gas. Consider that during the «breathing» of the planet, the temperature changes slightly, and C_V can be considered constant.

The secret planet

(3 points) In the evening, Hedgehog told Bear Cub about his discovery. In friend in his turn told him that he saw the Third planet which consists of a substance which caloric and thermal equations of state can be described as:

$$U = \alpha VT^4, \quad p = \frac{U}{3V},$$

where U is the internal energy, V is the volume occupied by the gas, T is the temperature of the substance, p is the pressure and α is a known constant.

«Psycho» was Hedgehog's thought. Why did he think like that?

First hint — 09.05.2022 14:00 (Moscow time)

Second hint — 11.05.2022 14:00 (Moscow time)

Final of the fourth round — 13.05.2022 22:00 (Moscow time)