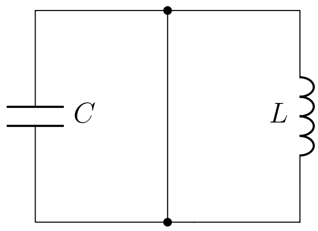
To hesitate no reason



Free vibrations with one degree of freedom

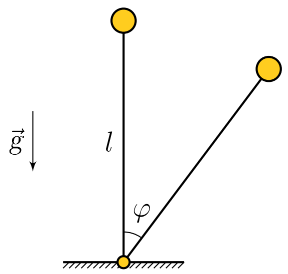
On a homogeneous weightless rod of length l in the gravity field there is a small load of mass m. At the base of the rod there is a hinge in which there is a returning moment of force, which is proportional to the angle of deviation of the rod $M = -k\alpha$, where angle α is the smallest angle measured from the vertical position of the rod, and k is a known positive coefficient.

1. *(1 pts)* What dimensionless parameter determines the nature of the movement of the system? Find how many times the frequency of small oscillations of such a pendulum differs from the frequency of small oscillations of a mathematical pendulum of length l for various values of the dimensionless parameter.

2. *(1 pts)* Depict all possible phase portraits of the trajectory of the motion of such a pendulum, i.e. dependence of $\dot{\varphi}$ on φ, where φ is the angle measured from the vertical position of the pendulum.

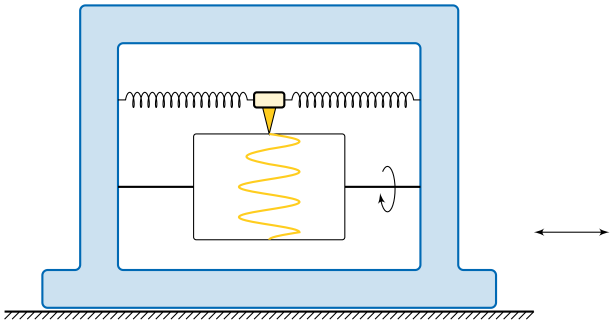
3. *(1 pts)* How does the dependence of potential energy on the angle of deviation of the pendulum change for various values of the dimensionless parameter?

The angle φ can take any value, the coefficient k can be considered given.



There are other problems on the next pages!

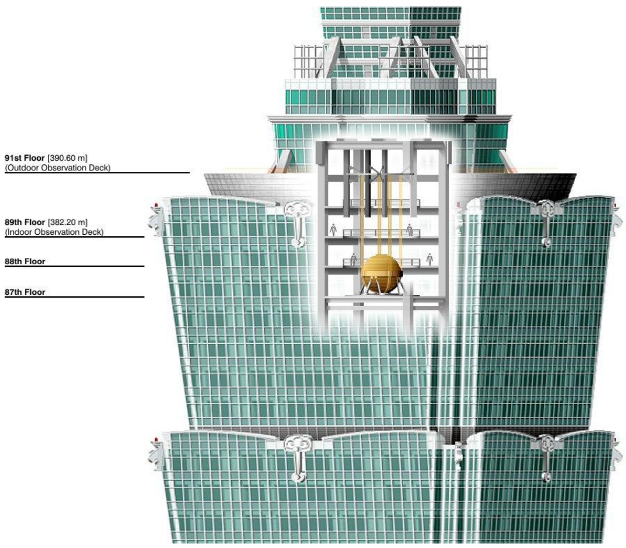
Forced oscillations with one degree of freedom

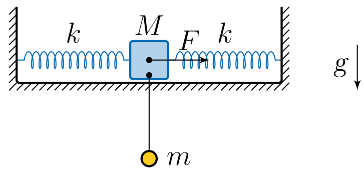


To record the horizontal vibrations of the foundation during an earthquake, you can use a device (vibrograph), the simplest diagram of which is shown in the figure. A weight of mass m is attached to the frame of the vibrograph using two stiffness springs k and a smooth drum rotating around the horizontal axis. The recording of vibrations on the drum occurs with a pen mounted on the weight.

(1 pts) Find the ratio of the amplitude of the harmonic vibrations of the foundation with the cyclical frequency ω to the amplitude of the forced oscillations of the weight recorded on the drum.

Forced oscillations with several degrees of freedom





It is known that the skyscraper « Taipei 101 », located in the capital of Taiwan — Taipei, is designed for use in natural conditions, where typhoons and earthquakes are not uncommon. In order to withstand strong winds and prevent lateral displacements of the skyscraper tower, it has the largest inertial vibration damper — a 660-ton steel pendulum in the form of a ball. To understand how a steel ball can reduce the amplitude of oscillations of a skyscraper tower, we propose to consider the simplest model discussed below. A bar of mass M, located on a smooth horizontal surface, is attached by two stiffness springs k to the walls, as shown in the figure. A small ball weighing m is suspended from it on threads with a length of l. A harmonic horizontal force $F(t) = F_0 \cos(\omega t)$ is applied to the bar.

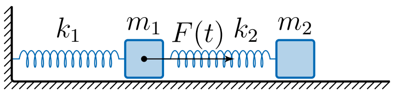
1. *(1 pts)* Find the dependence of the amplitude of small vibrations of the bar and ball in the steady state on the frequency ω.
2. *(1 pts)* Under what conditions is the amplitude of the forced oscillations of the bar equal to zero?
3. *(1 pts)* What is the amplitude of the ball vibrations in this case?

Viscous friction in the system is negligible. Gravity acceleration is g.

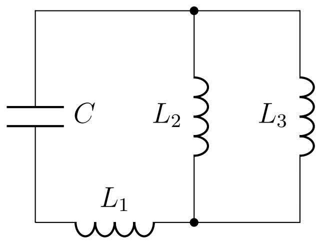
Electromechanical Analogies

It is well known that the mathematical form of recording vibrations in mechanical systems and electrical circuits has the same form. So, the equation describing the oscillation of a weight on a spring is no different from the equation of free oscillations in a LC circuit. In order to draw an analogy, it is enough to note that the equations describing the change in coordinate and charge are completely identical to each other, it is only necessary to replace the mass m with the coil inductance L, and the spring stiffness k by the reciprocal of the capacitor, $1/C$. Moreover, the presence of viscous friction is completely analogous to the inclusion of resistance in electrical circuits, it is only necessary to replace the coefficient of viscous friction with the resistance of the resistor. In this task, we suggest that you try to restore its mechanical analogue using a well-known electrical circuit and vice versa, try to obtain a similar mechanical system using a known electrical circuit.

1. *(0,5 pts)* Suggest a mechanical analogue to the serial RLC circuit with a constant emf source.
2. *(0,5 pts)* Suggest an electrical analogy to a mechanical spring pendulum located on a horizontal surface. Spring stiffness k, mass of bar m, coefficient of friction between bar and surface μ.
3. *(1 pts)* Suggest an electrical analogy to the next mechanical system. The surface is frictionless, $F(t) = F_0 \cos(\omega t)$



4. *(1 pts)* Suggest a mechanical system that will be similar to the electrical circuit shown in the figure



Authors: A.A. Kireev, L.M. Koldunov

First hint — 18.05.2020 14:00 (Moscow time)
Second hint — 20.05.2020 14:00 (Moscow time)

Final of the fourth round — 22.05.2020 22:00 (Moscow time)