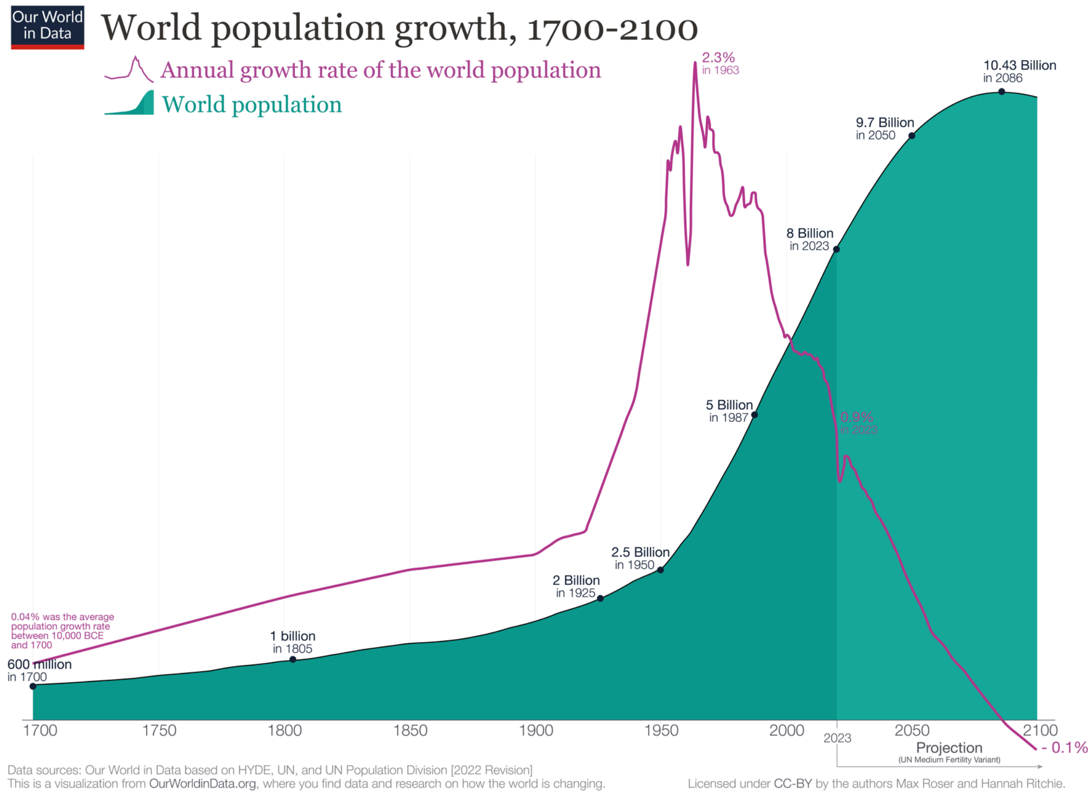
10.s05.e05

We are only as strong as we are united, as weak as we are divided.
Harry Potter and the Goblet of Fire

Synthesis

Part 1. Nuclear Reactions (2.5 points)

According to publicly available data, the 20 th century marked the beginning of rapid population growth on our planet.



1. *(0 points)* According on the graph, determine the year when the peak population growth rate occurred, i.e., the percentage of population growth for the year was maximum, and what this rate was in percent.
2. *(0.2 points)* Assuming that since 2023, the population growth rate will be equal to the peak growth rate in the 20th century, determine how many years later one person will stand on each square meter of land. Assume that the Earth's radius is 6400 km and the land area is 1/3.

Due to the rapid growth of the world's population, energy consumption is also growing rapidly. Today, humanity consumes approximately $Q = 10^{21}$ J per year. Renewable energy sources (wind, hydro, solar, etc.) can provide up to $3Q$ per year, but unfortunately, they do not solve all energy problems. The energy obtained with their help is quite expensive and requires additional conditions (wind availability, a large number of sunny days per year, etc.). The power generated by these sources varies greatly over time (for example, solar panels do not generate energy at night), which requires the creation of huge and expensive energy storage facilities.

Historically, humanity has learned to actively use natural resources: oil, coal, gas, etc. Unfortunately, their reserves are limited and unevenly distributed across the planet. For example, oil reserves are about $15Q$, coal reserves are about $150Q$. In addition, the burning of hydrocarbons leads to the emission of greenhouse gases and affects the climate.

In addition, chemical energy sources have low energy efficiency.

3. *(0.4 points)* Estimate **the energy efficiency** of burning chemical fuel, i.e., the amount of energy (in eV) per nucleon released during the combustion of one kilogram of coal ${}^{12}_{6}\mathrm{C}$. Assume that its specific heat of combustion is 31 MJ/kg.

For these reasons, humanity is searching for alternative energy sources. In the course of research in nuclear physics, it was found that the nuclei of some elements can be transformed into the nuclei of others. For example:

$$^{235}_{92}\mathrm{U} + {}^{1}_{0}n \longrightarrow {}^{144}_{56}\mathrm{Ba} + {}^{89}_{36}\mathrm{Kr} + 3\,{}^{1}_{0}n,$$

$$^{235}_{92}\mathrm{U} + {}^{1}_{0}n \longrightarrow {}^{140}_{54}\mathrm{Xe} + {}^{94}_{38}\mathrm{Sr} + 2\,{}^{1}_{0}n.$$

4. *(0.4 points)* Using the concept of mass defect, estimate the energy efficiency of each reaction.
5. *(0 points)* By how many orders of magnitude is the energy efficiency of nuclear fuel greater than the energy efficiency of chemical fuel?

Nuclear fission reactions often produce unstable isotopes, the decay of which results in a chain of nuclear transformations.

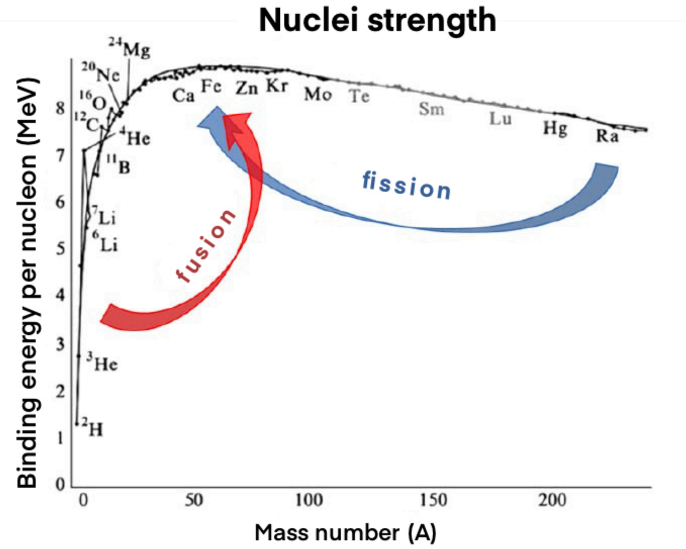
6. *(1.5 points)* Consider the possible decay chains of the nucleus ${}^{235}_{92}\mathrm{U}$, at the end of which the ${}^{207}_{82}\mathrm{Pb}$ nucleus is formed. Assume that only three types of reactions are possible: alpha decay, beta-minus decay, and proton emission. Assuming that their probabilities are related as $20 : 10 : 1$, respectively, determine the most likely set of decay products.

Note. In reality, proton emission is still an order of magnitude less likely and occurs with artificially created isotopes of chemical elements.

From all of the above, it follows that nuclear energy solves humanity's problems with energy supply. However, the operation of nuclear power plants leads to the accumulation of radioactive waste and the possibility of technological disasters, which forces us to continue the search for alternative energy sources

Part 2. Nuclear Fusion (0.5 points)

It has been established that energy can be released not only during the fission of heavy nuclei but also during the fusion of light elements from the beginning of the periodic table. A visual demonstration of this fact is shown in the figure, which depicts the dependence of the binding energy of nuclei on the atomic number. It can be seen that elements to the left of Fe tend to fuse, while those to the right tend to fission.



From a practical point of view, the following fusion reactions are of most interest:

$$
\begin{aligned}
d + d &\longrightarrow t + p + 4{,}03 \text{ MeV},\\
d + d &\longrightarrow {}^{3}_{2}\text{He} + n + 3{,}27 \text{ MeV},\\
d + t &\longrightarrow {}^{4}_{2}\text{He} + n + 17{,}6 \text{ MeV},\\
d + {}^{3}_{2}\text{He} &\longrightarrow {}^{4}_{2}\text{He} + p + 18{,}3 \text{ MeV},
\end{aligned}
$$

where d and t are the nuclei of deuterium and tritium, respectively. It is important to note that the first and second reactions are equally probable.

1. *(0 points)* Find the energy efficiency of the thermonuclear fusion reactions.

Unlike fossil fuels and nuclear fuel, deuterium is widespread in nature and is found in water. There is one deuterium atom for every 6500 hydrogen atoms.

2. *(0.5 points)* Estimate the minimum amount of gasoline that needs to be burned to obtain energy equal to that released in the fusion reaction of deuterium extracted from one glass of water (0.33 liters). The specific heat of combustion of gasoline is 44 MJ/kg.

For nuclear fusion reactions to occur, the Coulomb barrier of repulsion between interacting particles must be overcome. Therefore, for the implementation of controlled thermonuclear fusion, it is preferable to use hydrogen isotopes, which have the smallest charge among all other nuclei. However, even for them, thermonuclear fusion reactions can only proceed at very high temperatures, on the order of tens of keV (1 eV = 11604 K). At such temperatures, any substance transitions into a plasma state.

Part 3. Plasma (1 point)

Plasma is an ionized gas, i.e., a two-component system consisting of electrons and positively charged ions. To study the properties of this system, we will find the characteristic scales of distance and time.

Characteristic length scale

An important property of plasma is quasineutrality, i.e., the equality of positive and negative charge in any sufficiently large volume. The characteristic spatial scale in plasma is the distance at which a violation of quasineutrality is permissible.

To estimate such a distance, it is necessary to consider the screening of the potential of a point charge in a plasma. We will assume that the concentrations of electrons and ions obey the Boltzmann distribution:

$$n(r) = n_0 \exp\left(-\frac{q\varphi(r)}{kT}\right),$$

where n_0 is the normalization constant for both components, φ is the potential of the charge, q is its magnitude, and T is the temperature. **Remark.** In a plasma, the temperatures of ions and electrons may differ, but in this problem, we will assume that these components are in thermodynamic equilibrium. From the Zeroth Hint of the Third Episode, you know the Poisson equation, which has the form:

$$\Delta\varphi = -\frac{\rho}{\varepsilon_0},$$

where ρ is the charge density. In the one-dimensional case, this expression takes the form:

$$(\varphi)''xx = -\frac{\rho}{\varepsilon_0}$$

1. *(0.5 points)* Assuming that $q\varphi \ll kT$, dedimensionalize the Poisson equation and find the characteristic scale of charge separation.

Characteristic time in plasma

If the quasineutrality of the plasma is violated, the arising electric fields will tend to compensate for it.

2. *(0.2 points)* Consider a fully ionized plasma under normal conditions, in each cubic centimeter of which there are $7 \cdot 10^{16}$ electrons and ions. Suppose that there was a complete separation of charges: all the electrons gathered near one plane, and the ions near another. The distance between the planes is 1 cm. Then uncompensated charges will arise at the boundary of the region occupied by the plasma. Find the magnitude of the electric field arising in this layer.

3. *(0.3 points)* Find the time it takes for the charges to return to the equilibrium position.

When creating a fusion reactor, it is necessary to solve two main problems. The first is to confine the plasma with the required parameters for a sufficiently long time, and the second is to prevent the flow of heat and particles from damaging the reactor walls. Both of these issues are addressed through the use of magnetic fields and are analyzed in the 11th-grade problem.

In the 10th grade problem, we will get acquainted with the concept of plasma using the example of a self-sustained gas discharge, and also compare the process of maintaining the discharge and the criterion for reaching zero useful power of a thermonuclear reactor

Part 4. ~~Gorenje Criteria~~ Zero cycle (2.5 points)

This section challenges you to derive the criterion for achieving the output to zero useful power of a thermonuclear power plant.

Consider a plasma in some limited volume consisting of deuterium and tritium nuclei, as well as electrons formed during their ionization ~~(this has never happened and here it is again)~~ and having the same temperature as their entire environment ~~(not again, but once more)~~. Each type of nuclei behaves as an ideal gas with concentration n and temperature T. Deuterium and tritium undergo the fusion reaction $d + t \longrightarrow {}^4_2\text{He} + n + 17.6$ MeV.

We will consider a highly simplified model in which the collision cross-section of the nuclei does not depend on their velocity. That is, two nuclei undergo a fusion reaction if they approach each other at a distance less than the known sum of some effective radii $R = r_1 + r_2$. Assume that all reaction products, without interacting with the plasma, reach the reactor walls and transfer all their energy to it in the form of heat. The reactor walls, in turn, convert this heat into electricity, which is then used to heat/maintain the plasma temperature. This feedback system operates with an efficiency of η. It is known that if the feedback mechanism and nuclear reactions are stopped, the plasma will begin to cool down with a characteristic cooling time τ, which is determined by the design features of the reactor and is one of its key parameters.

1. *(2.5 points)* Under what circumstance on the value of $n\tau$ is it possible for the reactor to operate in such way that the described system can operate for an arbitrary amount of time and be "self-sustaining"due to the feedback mechanism?

Part 5. Discharges and the 3/2 Law (1.5 points)

Consider the simplest model of a vacuum diode. This device consists of two plates: a cathode and an anode placed in a hermetically sealed bulb in which a vacuum is created. When the cathode temperature is significantly increased, the process of thermionic emission begins — the emission of electrons from the cathode surface. When a voltage is applied to the anode, an accelerating (or decelerating, depending on the sign of the applied voltage) potential difference arises. As a result, a directed and ordered motion of electrons occurs i.e. an electric current. Let's study the dependence of this current on the anode voltage. We make the following assumptions: electrons start their motion from the cathode surface with zero initial velocity; the electric field strength at the cathode-vacuum boundary is zero; the plates can be considered equipotential, and their linear dimensions are much larger than the distance between them (the parallel plate capacitor approximation holds).

1. *(0.5 points)* Write down the Poisson's equation for an infinitely thin layer parallel to the planes of the plates, as well as the expression for the volume charge density for an arbitrary coordinate, depending on the concentration.
2. *(0.2 points)* Use the law of conservation of energy and obtain the relationship between the particle velocity and the potential at a given point in space.
3. *(0.3 points)* Find the potential of the electric field between the plates as a function of the coordinate.
4. *(0.5 points)* Obtain the dependence of the current on the anode voltage.

Part 6. Self-Sustained Discharge (2 points)

Let's qualitatively consider the process of the emergence of a self-sustained gas discharge.

The space between two flat conducting electrodes (cathode and anode), located at a distance d from each other, is filled with gas, and the electrodes themselves are connected to a DC voltage source.

The process of gas discharge formation consists of two parts. The first is the formation by electrons in the process of ionization of the current of electrons and ions in the medium filling the space between the electrodes. The second part is the knocking out of new secondary electrons by ions from the cathode.

Let's consider each of these processes in more detail.

Suppose that, for some reason, an electron is formed between the electrodes. Under the action of an electric field, it begins to accelerate and move from the cathode to the anode, colliding with gas molecules and ionizing them in the process. The new formed electrons and ions will also move towards the anode and cathode, respectively. At the same time, electrons will create more and more new electron-ion pairs, while ions will not (since their speed, which is one of the determining parameters in this process, will be significantly lower than the speed of electrons). For this reason, if nothing happens when ions hit the cathode, then after a while the process of current flow in the gas will stop, because all electrons will reach the anode, ions will reach the cathode, and new charges will not be formed.

However, when an ion hits the cathode, with a certain probability, electrons are knocked out, which again starts the process of electron-ion pair formation in the gas.

In this section, you are invited to explore this process.

Let's call the volume ionization coefficient α the number of electron-ion pairs formed per unit length of the electron path, and the coefficient of secondary ion-electron emission γ is the average number of electrons knocked out of the cathode by one ion. By external ionization, we mean some process that creates primary electrons. These can be a variety of processes: ultraviolet or γ radiation passing through the gas, an additional conductor creating a small current of electrons, and so on. In our review, we will assume that this process is localized in the cathode region.

1. *(0.3 points)* It is known that the electron current in the cathode region is I_{ek}, which includes the external ionizer current and the current of secondary electrons knocked out of the cathode by ions. The volume ionization coefficient α is known and constant throughout the volume between the electrodes. Find the electron current in the anode region.
2. *(0.3 points)* Determine the ion current in the cathode region I_{ik}.
3. *(0.7 points)* Express the cathode and anode electron currents I_{ek} and I_{ea} in terms of the ionization current I_i.

Breakdown is a situation where the electron current I_{ea} grows infinitely with an arbitrarily small ionization current. If, after the breakdown occurs, the external ionizer is removed and the current does not stop, then the discharge is called self-sustained: ionization is maintained by processes in the discharge itself.

4. *(0.7 point)* Determine under what circumstances on α, γ, and d breakdown will occur and the discharge will become self-sustained.

Note. You might find useful the following fact:

$$(e^{\alpha x})' = \alpha e^{\alpha x}.$$

First hint — 29.05.2024 20:00 (Moscow time)
Second hint — 31.05.2024 12:00 (Moscow time)

Final of the fifth round — 02.06.2024 18:00 (Moscow time)