LPR V Cup

10.s05.e04

Nothing is as invigorating as a meteorite in the morning
Folk wisdom about the Chelyabinsk meteorite

Αστρον βλημα

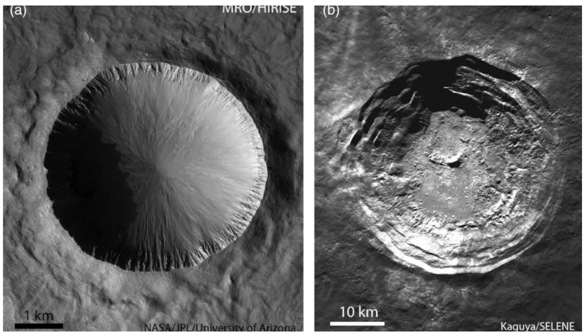
"Fog" - thought Hedgehog and felt like home.

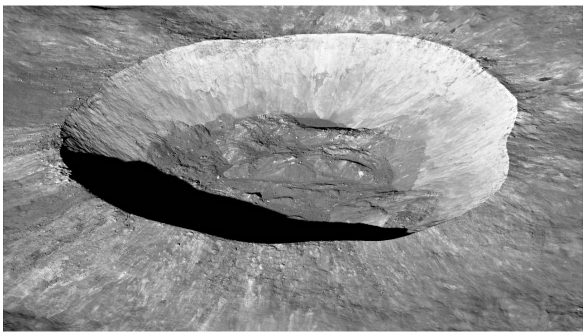
He liked the island immediately, and he decided to separate from the group for a short time and wander in the synoptic conditions that were natural for him. He walked further and further, going deep into the island, where the fog became thicker and thicker, and the cries of the seagulls became quieter and quieter. And at some point, it is not known exactly which one, because Hedgehog, as always, lost track of time, he almost stumbled upon a weakly trodden path. Hedgehog was delighted with it, because besides time, he had already lost his way as well. And even the compass, presented by one of the gentlemen, could not help him, because its needle rather phlegmatically walked in a circle, as if in childhood it dreamed of becoming a second hand, and not at all an arrow of a compass.

Hedgehog walked along the winding path and imagined that it was not it which twisting so much in bizarre zigzags, but the world around. He walked for a long time, for a very long time, but at some point he noticed that the fog began to dissipate, and the compass needle began to calm down. It no longer spun as if it were the hand of a clock, but all the time showed the same direction - along a winding path. Hedgehog continued on and soon, in a faint haze of fog, he saw the dome of the observatory. Never in his life had he been as happy as he was at that moment. His paws began to jog, and the observatory, which did not loop as the world around it approached faster and faster.

There was no one inside the observatory, but it was very cozy. The first thing Hedgehog did was looking through the telescope and saw that it was tuned to some distant planet, the surface of which was decorated with craters of various sizes.

He looked at the table, and next to a cup of still hot tea he saw black and white photographs of craters and neat calculations on sheets of yellowed paper. Hedgehog climbed onto a high stool with difficulty and began to study them carefully.





Part 1. Crater volume

This paper studies the process of impact crater formation. To build a model, consider the collision of a meteorite of mass m and density δ with a planet which surface is represented by rocks with density ρ and strength Y (Pa). Meteorites fly at a speed u perpendicular to the surface. Acceleration of gravity is g.

In this model, the volume of the crater V depends on the 6 parameters given above:

$$V = V(m,\ u,\ \delta,\ Y,\ \rho,\ g)$$

1. *(1 point)* Using the dimension method, suggest 4 independent dimensionless combinations, including V, m, u, δ, Y, ρ, g. In this case, each of the parameters V, ρ, g, Y must set its own separate combination in which it is included in the first degree and does not appear in other combinations.
2. *(1 point)* Let's use the «extended» dimension method, i.e. distinguishing units of length by coordinates. For example, volume $[V] = [L_x][L_y][L_z]$, and pressure $[p] = [F][L_x]^{-1}[L_y]^{-1}$, where $[F]$ is the dimension of normal pressure force. Applying this method and instructions from point 1, get all dimensionless combinations of dimensional quantities. To what extent is a dimensionless combination containing the value g included?
3. *(0 points)* Give a physical interpretation of each dimensionless combination from points 1 and 2.
4. *(1 points)* Using the results of point 2, obtain the dependence of the crater volume V on the parameters $(m,\ u,\ \delta,\ Y,\ \rho,\ g)$ up to a multiplier.

Hedgehog snorted with delight and began to study in more detail photographs of craters and records of the meteorites that created them. After a while, he collected enough data to continue the research of the Unknown Author.

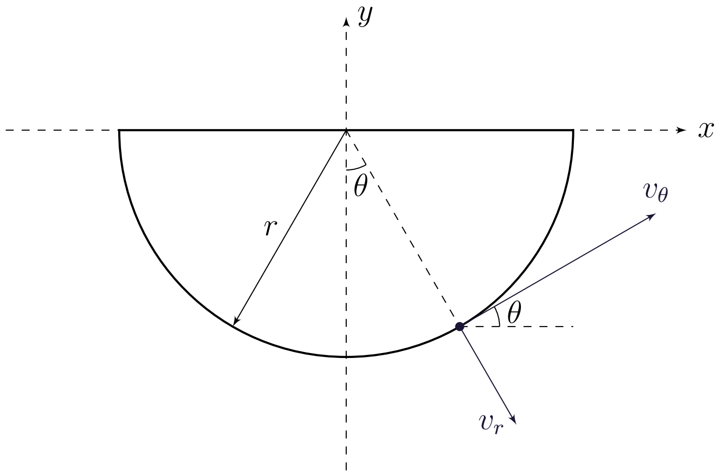
5. *(3 points)* Using the table compiled by Hedgehog, determine the exponents in the equation obtained in point 4. Hedgehog selected meteorites of approximately the same size, and craters in the form of spherical holes with a cross-sectional radius on the planet's surface of R and a depth of h.

№	R, m	h, m	δ, kg/m^3	ρ, kg/m^3	u, km/s	Y, MPa
1	804	131	4270	3122	9	9,9875
2	307	43	4500	3290	2	10,5255
3	1800	490	4472	3270	51	10,46
4	7800	810	4308	3150	199	10,0766
5	926	132	4904	1477	9	11,4705
6	813	115	2000	1972	9	4,678
7	1311	259	2000	1477	12,4	10
8	19989	2780	3565	1880	250	56,7009

Part 2. Rock fragmentation velocity

In the notes of the Unknown Author the velocity of rock fragments immediately after impact was also studied. The origin was placed at the point of impact of the meteorite on the planet's surface and the fragments which are formed on a sphere of radius r centered at the origin were considered. It is known that the initial radial velocity of the fragments equals $v_r = A/r^m$, где A is a dimensional constant, and the degree is $m \in (2; 3)$.

1. *(3 points)* Assuming that the scattering of fragments can be described by the model of an incompressible fluid, find $v_\theta(r, \theta)$.
2. *(1 point)* Neglecting the influence of gravity at the initial stage, determine the equation for the trajectory in polar coordinates $r(\theta)$.



If you are participating for 2 grades (9th and 10th), you need to submit the work in the Google class only of the 10th grade, having sent all the points of both problems. If you first solve a problem for the 9th grade, then the items of the 10th grade problem that are not sent will be evaluated as incorrect.

First hint — 20.05.2024 20:00 (Moscow time)
Second hint — 22.05.2024 12:00 (Moscow time)

Final of the forth round — 24.05.2024 20:00 (Moscow time)