LPR V Cup

10.s05.e01

Hint 2

IMPORTANT! The next task is both a hint and an alternative to the main task. Three important points:

1. You can continue to send the solution to the main problem.
2. At any moment before the final deadline you can start to solve the Alternative problem. If you do so, at the beginning of the solution write: *I am doing the Alternative problem!* In this case a penalty coefficient for the Alternative problem is

$$0{,}7 \cdot \sum_i \frac{k_i \cdot p_i}{10},$$

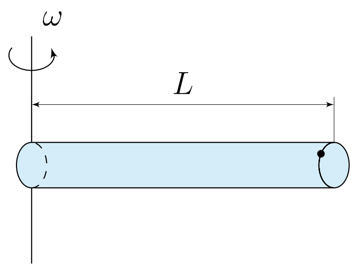
where p_i is a point for the problem item, and k_i is a penalty coefficient for the corresponding problem's item at the moment of moving to the Alternative problem. In other words, maximal points for the alternative problem equals to the maximal points you can gain at the moment of moving to the alternative one multiplied by 0,7. Also, we remind you that a penalty coefficient can't be less than 0,1. Solutions of the main problems from that moment will not be checked. Be careful!

3. The task consists of several items. The penalty multiplier earned **before** is applied to all points. In the future, each item is evaluated as a separate task. If you send a solution without any item, this item's solution is considered as Incorrect. For more information about scoring points for composite tasks, see the rules of the Cup.

Alternative task

Part 1. Ant and the centrifuge

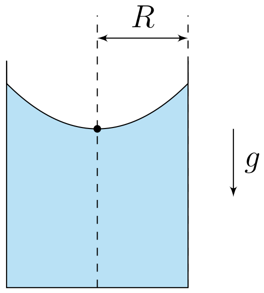
Ant is inside a cylindrical flask with a length L at one of its circular bases. The flask was spun around a vertical axis passing through the opposite base with an angular velocity ω.



1. *(1 point)* At what minimum velocity does Ant need to push off from the base to get to the middle of the flask? The inner walls of the flask are smooth, Ant's collisions against the walls are completely elastic.

Part 2. Ant and a cylindrical vessel

Ant fell into a cylindrical glass of radius R filled with water and in order to save it (Ant, not the glass), the vessel with water was spun around its vertical axis with an angular velocity ω, and after a long time it was instantly frozen (water, not Ant). Ant woke up in the center of the formed ice surface and started trying to escape from it.

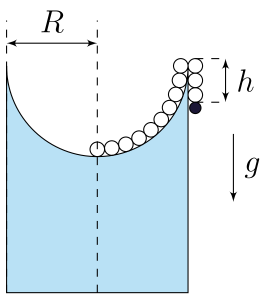


1. *(1 point)* What is the minimum velocity in the horizontal direction that needs to be given to Ant so that he succeeds? The surface of the ice is absolutely smooth. The radius of the vessel is R. Accept the acceleration of gravity g.

Assume that the bottom of the glass is covered with water and the liquid reaches the top edge of the glass.

Part 3. Caterpillar's transmission

Ant realized that he wouldn't be able to get out of this situation on his own. So he called for help from his friend Caterpillar ~~Mr. Wolfe~~. They had a plan and they followed it.



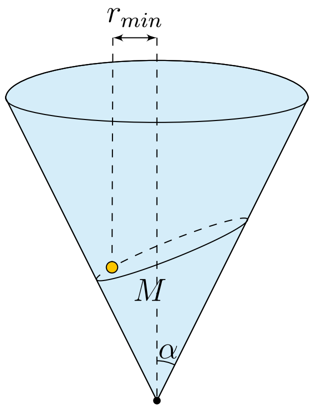
First of all they formed up the ice surface to become a half-sphere of radius equal to the radius of the vessel R. After that Caterpillar partially hung over the edge of the vessel as shown in the figure (the length of the hanging part of the caterpillar is h) and Ant hung on her tail so that they were in balance.

1. *(2 points)* Considering Caterpillar to be uniform along its entire unknown length L, determine the ratio of the mass of Ant to the mass of Caterpillar.

In the figure Ant is indicated by a black circle. The Caterpillar's head is located at the bottom of the ice surface.

Part 4. Ant in Glamour

Ant was fulfilled with sliding on various kinds of surfaces and crawled into the already familiar cone-shaped bag with angle α, made of glossy paper. He pushed off and began sliding along a closed trajectory so that the minimum distance to the axis of the cone during movement is r_{min}. The mass of the ant is m. Determine:

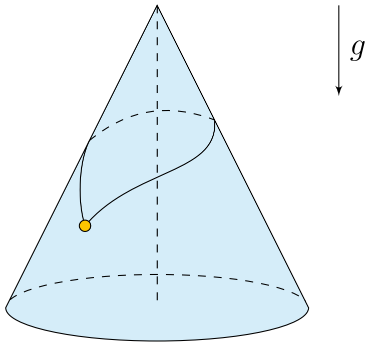


1. *(0,5 points)* The gravitational torque at the moment of maximum approach to the axis of the cone relative to its apex.
2. *(0,5 points)* Projection of the total moment of all forces applied to the ant on the axis of the cone at the moment of maximum approach to it.

Gravitational acceleration is g. The base of the cone is parallel to the horizontal surface.

Part 5. Local Alps

Ant the Traveller saw another, much larger bag in the distance and decided to try himself at mountaineering. Armed with a weightless thread, he set off to conquer a cone made of smooth glossy paper. As a result of a long ascent, he found himself hanging on the surface of the cone, holding on to the two free ends of the thread (see figure). The mass of the ant is m, the length of the thread is L, the half-angle of the cone is α, the acceleration of gravity is g. Determine:

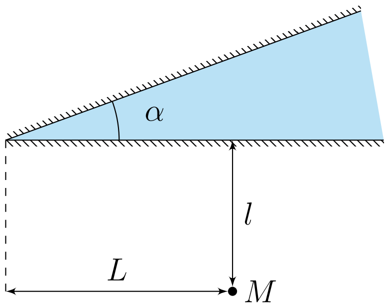


1. *(0,5 points)* Thread tension force T.
2. *(0,5 points)* The angle φ between the two ends of the thread attached to the ant.
3. *(1 point)* All possible values of the angle α.
4. *(0 points)* If the thread would change its shape if it had a constant linear density.

Part 6. How not to get into a trouble puddle

Along the journey Ant ended up next to a puddle and wanted to cross it. On land, Ant runs with a speed v, and his she-friend, a Water Strider, can drive him through a puddle with a speed $v/\sqrt{2}$.

The angle α is equal $30°$. The geometric parameters are shown in the figure. It is known that $L = 2l$. At the initial moment of time, Traveler-Ant is located at the point M.



1. *(0,5 points)* In what time will he reach the opposite «shore» of the puddle if he always moves perpendicular to the next intersected boundary?
2. *(0,5 points)* In what time will he reach the opposite «shore», if his velocity is always directed at an angle $60°$ to the nearest edge of the puddle, so as to approach the vertex of the angle while moving?

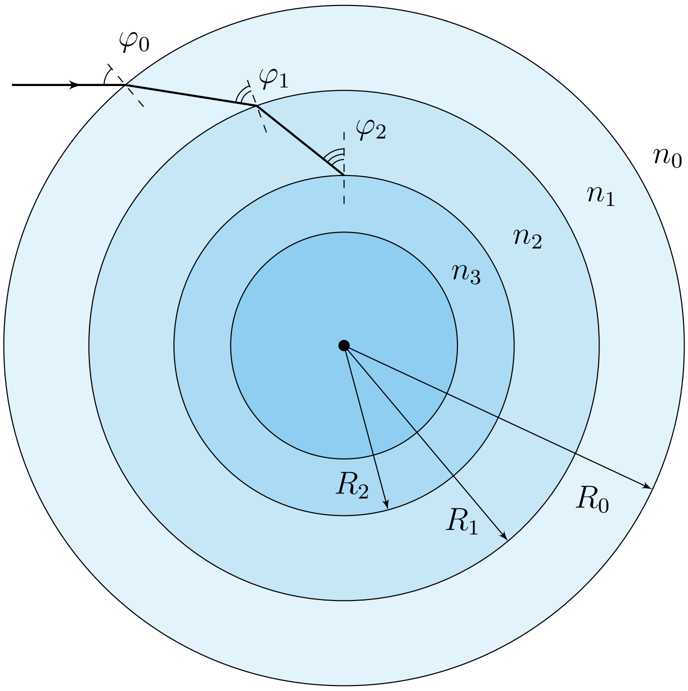
3. *(1 point)* What is the minimum possible time it takes for Ant to move the opposite «shore»?

Assume that the distance l is known. In paragraph 2 the angle is specified relative to the horizontal border.

Part 7. Sphere

A ray of light falls at an angle $\varphi_0 = 45°$ on an optical system consisting of concentric spheres of different radii and different refractive indeces. The radii of the spheres are $R_N = R/2^N$, and the refractive indices are $n_N = n_0 \cdot 2.5^N$, where N is the number of the sphere (see figure). Find:

1. *(1 point)* the angle of incidence of the ray φ_{34} when passing through the interface between media 3 and 4?



Mathematics software may be useful for you to solve some of the items. Numerical answers must be presented with an accuracy of at least 1%.