LPR Cup

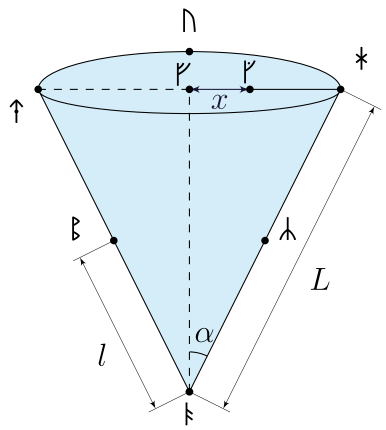
10.s05.e01

Born to crawl will rise to the sky
Ant-Man

Pilgrim Ant

Part 1. Kinematics

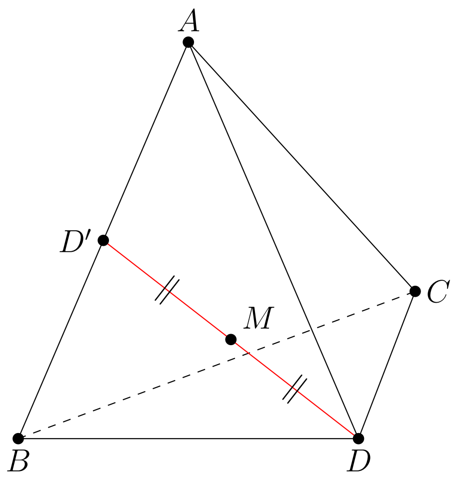
Traveler – Ant can crawl along the lateral surface of a newspaper bag in the form of a right circular cone without a base. The angle of the cone is α.



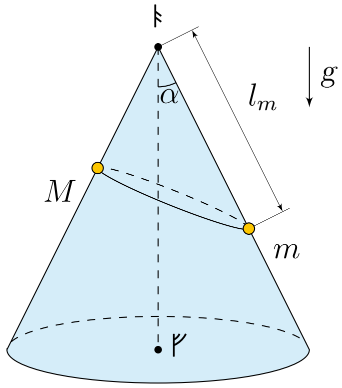
Find:

1. (*1 point*) Ant got carried away reading the newspaper from which the bag was made and noticed an interesting letter $\mathfrak{H}$ at a distance of $l/2$ from the letter ᚦ. He began to move towards it in such a way that his speed began to change according to the law $v(r) = a/r$, where a is an unknown constant, and r is the distance to the letter ᚦ. Ant really wanted to reach it as soon as possible, so he chose such a trajectory to get to the letter $\mathfrak{H}$ in the shortest possible time without losing sight of it, i. e. without making a single complete turn around the bag. Find the angle between the velocity vector of Ant at the beginning of the path and ᚦᛏ, if as it approached the letter $\mathfrak{H}$ it moved parallel to the base of the cone.

2. (*2 points*) Ant was blown away by a sharp gust of wind from the bag and when the wind died down, it found itself on a milk carton in the form of a regular tetrahedron. When it regained his sences, he found that he was sitting on the letter M, which turned out to be the middle of the height of DD'. To get a better look at the setting sun, Ant decided to run to the edge of AC and, out of professional habit, he wanted to do it in the minimal time. Ant crawls along the face of ABD at a speed of v, and on the faces of ACD and ACB live his old friends Caterpillar-Surveyor and Haymaker-Spider respectively, who are always ready to give a ride to the Traveler Ant on their face. The speed of Caterpillar is $\sqrt{3}v$, and the speed of Spider is 10,2v. Since the milk carton is on the ground, Ant cannot move along the face of BCD. What is the minimum time it takes to get from the letter M to the edge AC? The length of the edge of the tetrahedron is a.



Part 2. Statics

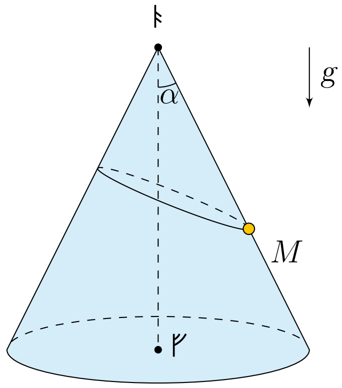


One sunny day Haymaker-Spider and Traveler-Ant found a bag made from a glossy Vogue magazine. It had a form of a right circular cone with an determined angle of α and was located so that its axis ᚴᚠ was aligned vertically. The free fall acceleration is g.

Glossy magazines are smooth, and in order to stay on the bag, Spider wove two weightless inextensible identical threads of length l and with their help hung together with Ant on the cone as shown in the figure. It is well known that the web of Haymaker-Spider is not sticky, so there is no friction between it and the bag.

Considering the masses of Ant and Spider equal to m and M, respectively, determine:

1. (*0,5 points*) The tension force of the threads T_0 in the equilibrium position. In this item the masses of Spider and Ant are the same, and the angle $\alpha = \pi/6$;
2. (*1 point*) at what distance l_m from the vertex ᚴ will Ant be located in a stable equilibrium position if the insect masses differ;
3. (*1 point*) The tension forces of the threads T in the equilibrium position;
4. (*0,5 points*) At what angles α is such an equilibrium possible.



When Ant ran home, Spider wove a massive homogeneous inextensible thread of unknown length and constant thickness and hung with it on a bag (see Fig). The ends of the thread are attached to the insect. It is known that the minimum and maximum distances from various points of the thread to the vertex ᚴare $l_{\min} = l_0$ and $l_{\max} = 2l_0$, respectively, where l_0 is an unknown quantity. The linear density of the thread is $\lambda = M/l_0$. Determine:

5. (*2 points*) The maximum tension force of the thread $T_{\max}$

In the wind, the bag, together with Spider, spun around the vertical axis ᚴᚠ, so that this mechanical system rotates with a certain constant angular velocity ω. It is known that the distance from any point of the thread to the tip of the cone ᚴ does not change during movement, and the minimum and maximum distances are $l'_{\min} = al_0$ and $l'_{\max} = bl_0$, respectively, and $a \neq b$, and $\omega^2 = 2fg/l_0$, where a, b, f are known constants. The thread and Spider do not come off the surface of the bag. Determine:

6. (*2 points*) How much do the maximum and minimum thread tension forces differ?

First hint — 29.04.2024 20:00 (Moscow time)
Second hint — 01.05.2024 12:00 (Moscow time)

Final of the first round — 03.05.2024 20:00 (Moscow time)