



## LPR Cup 2023

10.s04.e04

### Hint 2

**IMPORTANT!** The next task is both a hint and an alternative to the main task. Three important points:

1. You can continue to send the solution to the main problem.
2. At any moment before the final deadline you can start to solve the Alternative problem. If you do so, at the beginning of the solution write: *I am doing the Alternative problem!* In this case a penalty coefficient for the Alternative problem is

$$0,7 \cdot \sum_i \frac{k_i \cdot p_i}{10},$$

where  $p_i$  is a point for the problem item, and  $k_i$  is a penalty coefficient for the corresponding problem's item at the moment of moving to the Alternative problem. In other words, maximal points for the alternative problem equals to the maximal points you can gain at the moment of moving to the alternative one multiplied by 0,7. Also, we remind you that a penalty coefficient can't be less than 0,1. Solutions of the main problems from that moment will not be checked. Be careful!

3. The task consists of several items. The penalty multiplier earned **before** is applied to all points. In the future, each item is evaluated as a separate task. If you send a solution without any item, this item's solution is considered as Incorrect. For more information about scoring points for composite tasks, see the rules of the Cup. **Since switching to an alternative selection, there is no opportunity to return to solving the main task.** Also, after switching to an alternative task **the points for the main task are reset.**

## Alternative problem

**Task 1.** (0 points) Find the difference  $\tan x - \sin x$ , assuming  $x \ll 1$ .

**Task 2.** (2 points) Seeker is on a tower of height 40 m. Find the distance from Seeker to the horizon where Hidden is. What is the difference between this distance and the distance that he will have to walk along the Earth. The radius of the Earth is 6400 km.

**Task 3.** (0 points) An elastic tourniquet with stiffness  $k$  is stretched from an undeformed state by  $\Delta x$ . Calculate the work of the elastic force using the definition of work of a force.

**Task 4.** (0 points) The rod of mass  $m$  and length  $l$  is located on a rough horizontal surface. The coefficient of friction between the surface and the rod is  $\mu$ . The rod begins to rotate around a vertical axis passing through one of its ends. Find the torque of friction force relative to the fixed end of the rod.

**Task 5.** (0 points) Find the work carried by one mole of an ideal gas with an isothermal volume increase of two times.

**Task 6.** (0 points) A rubber tourniquet in the form of a ring of initial radius  $r_0$  and stiffness  $k$  is put on a vertical cylinder of radius  $R > r_0$ . The coefficient of friction between the harness and the cylinder is  $\mu$ . The plane where the tourniquet is located is horizontal. Acceleration of gravity  $g$ .

1. What is the maximum mass of the ring  $m_{\max}$ , at which it does not fall along the cylinder?
2. What is the pressure that the ring exerts on the cylinder? The thickness of the ring is  $\delta$ .
3. If the mass of the ring is  $m = m_{\max}/2$ , then to what angular velocity should the cylinder be unwound so that the ring slides off the cylinder?

**Task 7.** (2 points) A point llama weighing  $m = 400$  g is suspended on two rubber cords that have been cut off from the same rubber cord. The length of the smaller piece of rubber in the unstretched state is  $l = 20$  cm. The stiffness of the cords is equal to  $k_1 = 30$  N/m and  $k_2 = 50$  N/m. At what distance from the ceiling is the llama hanging? Give the answer in centimeters. If necessary, round to tenths. How will the answer change if the llama's mass is 800 g.

**Task 8.** (0 points) The load weighing  $m = 5$  kg is suspended from the ceiling on an elastic rubber cord with a stiffness of  $k = 500$  N/m. The initial velocity directed vertically upwards is transmitted to the load twice. In the first case, this velocity is equal to  $v_1 = 0,5$  m/s, and in the second —  $v_2 = 2$  m/s. How many times is the maximum lifting height of the load, counted from the starting point, in the second case greater than in the first?

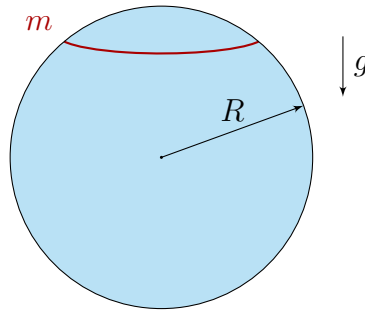
**Task 9.** Solve the equations numerically:

$$\sin x = \frac{1}{3}x, \quad 2x = \ln x + 5.$$

**Task 10.** An elastic tourniquet in the form of a ring of radius  $r_0$ , stiffness  $k$  and an unknown mass  $m$  is placed on a smooth ball of radius  $R = 2\sqrt{2}r_0$ .

1. (2 points) Find all possible values of the mass of the ring for which there is one equilibrium position.
2. (2 points) How many equilibrium positions are possible in this system?

The plane of the ring is horizontal. Acceleration of gravity  $g$ .



**Task 11.** (2 points) A weightless spring with stiffness  $k$  in an undeformed state has a length  $R_0$ . It was fixed at the upper point of a smooth sphere of radius  $R > R_0$ , and a point mass  $m$  was attached to the lower end. Find the equilibrium position of the system. Is it possible for the system to have two equilibrium positions?

