LPR Cup 2023

10.s04.e04

A diamond is a piece of coal that
has successfully survived stress.
Folk wisdom

Pavel Shishkin's rough talents

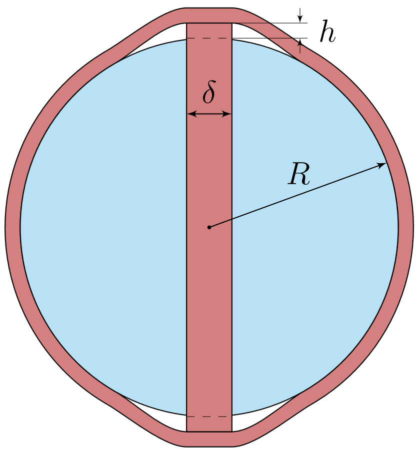
Pavel Shishkin decided to come up with the problem for the experimental tour of regional contest on Physics and make it dazzling. He found a graphite ball of radius R_1 and an unlimited number of stationery elastic bands in the form of rings of a known radius, width δ and thickness h in the 6^{th} dormitory of MIPT. The stiffness of all elastic bands was equal to k and **did not change under any stretching**. He set himself the goal of making a diamond out of a graphite ball. To do this, he began to compress the ball with stationery elastic bands using the following technology:

- Each rubber band makes only one turn and has no self-intersections.
- The radius of the stretched elastic band coincides with the radius of the layer on which it is stretched and must lie in the diameter plane of the ball (see Figure).
- The elastic bands are put on so that the shape can be considered a ball within the error of the experiment.
- The effects of gravity and friction may be neglected.

Experiment 1. Preparational

Elestac band has width δ and thickness h, radius of the ball is R. Pavel put on the first Elestac band, and then the second one so that they are perpendicular to each other.

1. *(1 point)* Estimate the difference between deformations of the elastic bands considering that their initial radius $R_0 < R$, assuming $h \ll R$ and $\delta \sim h$.
2. *(Pavel Sishkin's respect)* Find the volume of a the cavity caused by the intersection of elastic bands and the relation between δ, h and R at which the total volume of such cavities might be neglected.



In next experiments assume that

- Pavel is such a talented experimenter that the air volume inside the elastic bands' ball might be neglected regardless of the linear dimensions.
- By compression the volume of the ball is changed obeying the law

$$\alpha_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T,$$

  where $\alpha_T = 3 \cdot 10^{-11}$ Pa^{-1} is the coefficient of compression in experiments, V_0 is the initial volume, V and P are the current volume and pressure exerted on it.
- The pressure of phase transition graphite-diamond at room temperature 300 K is about 10 GPa.

Experiment 2. Heat-conducting rubber bands of a fixed cross-sectional area

As part of this experiment, Pavel selected those rubber bands that did not change their cross-sectional area during deformation and conducted heat well. The radius of these rubber bands in the undeformed state was $R_0 = 80$ cm, width $\delta = 1$ cm, thickness $h = 40$ microns, stiffness $k = 400$ N/m, radius of graphite ball $R_1 = 1$ m.

3. *(4 points)* How many rubber bands will Pavel Shishkin need to turn graphite into diamond?

Experiment 3. Heat-conducting rubber bands of fixed volume

Pavel continued his experiments in the 6th dormitory of MIPT and selected those rubber bands that did not change their volume during deformation and also conducted heat well. The radius of this type of rubber bands in the undeformed state was $R_0 = 95$ cm, width $\delta = 1$ cm, thickness $h = 40$ microns, stiffness $k = 400$ N/m, radius of graphite ball $R_1 = 1$ m.

4. *(4 points)* How many rubber bands will Pavel Shishkin need to turn graphite into diamond?

Experiment 4. Heat-insulated rubber bands

But even this was not enough for Pavel. He selected those rubber bands that did not conduct heat and performed the same experiment.

5. *(1 point)* Find the analytical dependence of pressure on temperature in this process, if the Pavel's goal is to reach the triple point ($p = 10.8$ MPa, $T = 4600$ K).
6. *(Respect of Щебень)* What is the initial temperature in the 6th dormitory of MIPT at atmospheric pressure so that Pavel Shishkin's graphite hits the triple point ($p = 10.8$ MPa, $T = 4600$ K).

When graphite is compressed, the total work of external and internal forces can be written as

$$\delta A_{\text{sum}} = \beta_V PTdV, \qquad \beta_V = \frac{1}{P}\left(\frac{\partial P}{\partial T}\right)_V,$$

where β_V is the isochoric pressure coefficient. Consider that the heat capacity of graphite is constant and equal to 2.1 kJ/(kg · K), the coefficient is equal to that.

NB. 1. Tabled values might be searched with the help of service. **2.** At the end of the experimental tour, you can eat the resulting diamond.

First hint — 15.05.2023 20:00 (Moscow time)
Second hint — 17.05.2023 12:00 (Moscow time)

Final of the fourth round — 19.05.2023 20:00 (Moscow time)