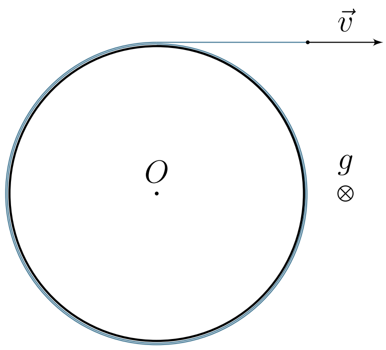
All energy is only borrowed, and
one day you have to give it back.
Avatar (movie)

Ring on a scabrous surface

In mechanics, you often encounter principles that are consequences of more general statements. Such a principle, for example, is the extremal total potential energy principle, which states that in the equilibrium position the potential energy of mechanical energy takes an extreme value.

Within this task, you are invited to apply another, not so well-known principle — the minimum energy dissipation principle, which has found its main purpose in hydrodynamics, to study the dynamics of a mechanical system.

The examined mechanical system is a ring on which an always-tense weightless inextensible thread is wound, the free end of which moves along its free rectilinear section (see the Fig.).

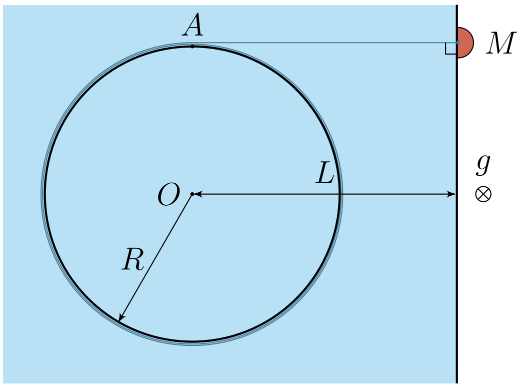


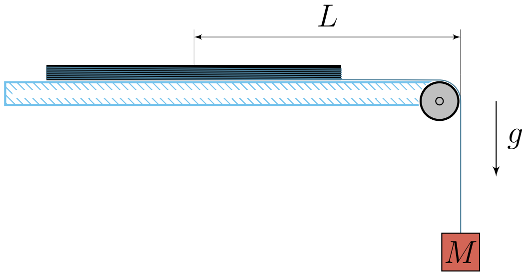
The ring is located on a flat scabrous horizontal surface. The coefficient of friction between the ring and the surface is equal to μ The mass m of the ring and the normal force acting from the surface on the ring are uniformly distributed along its perimeter. Gravitational acceleration is g.

To find the minimum possible tension force T_{min} of the free part of the thread, at which the initially stationary ring begins to move, you can do the following: if the kinetic energy of the ring is constant, then the power of the forces acting on it must be zero. So, the problem can be reduced to finding the minimum possible dissipation power caused by the friction force of the ring on a scabrous surface at a fixed value of the velocity v of the free end of the thread.

1. *(4.5 points)* Find the minimum possible tension force T_{min} of the free part of the thread at which the ring begins to move.

Let's take a look at the mechanical system shown in the figures below. A ring of mass m and radius R lays on a horizontal table, the center O of the ring is located at a distance $L > R$ from the straight edge of the table. A weight of M is attached to the free end of the thread.





When stretched, the free section of the thread has three sections:

- a horizontal section, oriented perpendicular to the edge of the table;
- a section where the thread touches a small weightless pulley;
- vertical section.

The weight is lifted to a height H above the position in which the free section of the thread is stretched, and released without initial velocity.

We will call "the process of absolutely inelastic deformation" such an infinitesimal deformation of the free section of the thread, in which the equality of the velocities of the weight and the point A of the ring at which it touches the free section of the thread is always maintained after it is reached.

2. *(2 points)* Find the velocity v_0 of the center of the ring immediately after the process of absolutely inelastic deformation of the free section of the thread.
3. *(3.5 points)* At what minimum coefficient of friction $\mu_{\min}$ between the ring and the table in the process of movement, the ring will stop, remaining entirely on the table?

First hint — 08.05.2023 20:00 (Moscow time)
Second hint — 10.05.2023 12:00 (Moscow time)

Final of the third round — 12.05.2023 20:00 (Moscow time)