Hint 2

IMPORTANT! The next task is both a hint and an alternative to the main task. Three important points:

1. You can continue to send the solution to the main problem.
2. At any moment before the final deadline you can start to solve the Alternative problem. If you do so, at the beginning of the solution write: *I am doing the Alternative problem!* In this case a penalty coefficient for the Alternative problem is

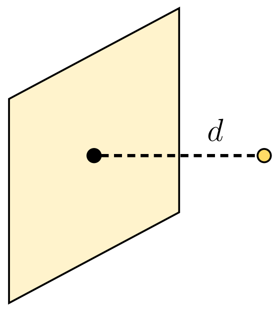
$$0{,}7 \cdot \sum_i \frac{k_i \cdot p_i}{10},$$

where p_i is a point for the problem item, and k_i is a penalty coefficient for the corresponding problem's item at the moment of moving to the Alternative problem. In other words, maximal points for the alternative problem equals to the maximal points you can gain at the moment of moving to the alternative one multiplied by 0,7. Also, we remind you that a penalty coefficient can't be less than 0,1. Solutions of the main problems from that moment will not be checked. Be careful!
3. The task consists of several items. The penalty multiplier earned **before** is applied to all points. In the future, each item is evaluated as a separate task. If you send a solution without any item, this item's solution is considered as Incorrect. For more information about scoring points for composite tasks, see the rules of the Cup.

The main task there are several possible ways of solving. We do not know which path you will eventually take, so we offer you several tasks, some of which may help you come to the cherished `Correct`. Do not send us solutions of the examples!

Study these materials.

Example 1. At the distance of 10 cm from the point charge, there is a uniformly charged square plate with dimensions of 20 cm × 20 cm as shown on the picture (the charge is placed on the continuation to the normal to the center of the plate). How much will the interaction force between the plate and the point-charge change, if the charge of the plate is concentrated at the center of the plate? How will the answer change, if the dimensions of the square are much greater than the distance from its center to the charge.

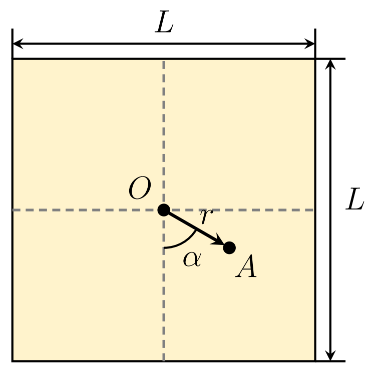


Answer. $\dfrac{E_2}{E_1} = \dfrac{6}{\pi}$; $\dfrac{E_2}{E_1} = \dfrac{2}{\pi}$.

Example 2. Three adjoined faces of the Cube are uniformly charged with a charge density of $+\sigma$, and the others have a charge density of $-\sigma$. Find the electrical field strength E at the center of the Cube.

Answer. $E = \dfrac{\sigma}{\varepsilon_0\sqrt{3}}$.

Example 3. A thin dielectric square plate with a side length L is uniformly charged with a surface density of $\sigma > 0$. Point A is offset relative to the plate center O in a plane of the plate for the small distance $r \ll L$ at $\alpha = 60°$ angle to the side of the square (see pic.). At that point, there is a small smooth dielectric puck placed with a mass m and a charge of $q < 0$. The puck is released with zero initial velocity. Determine the magnitude and direction of the puck acceleration right after the release. After what time the puck will occur at the minimum distance from the center of the plate for the first time? Ignore the gravity force. The plate is fixed.



Answer. $a = \dfrac{\sigma q r\sqrt{2}}{\pi\varepsilon_0 m L}$; $t = \sqrt{\dfrac{\pi^3\varepsilon_0 m L}{4\sqrt{2}\sigma q}}$.

Alternative problem

1. *(3 points)* A thin dielectric square is uniformly charged at its perimeter with a known linear density of the charge λ. Find the field strength on the axis that is perpendicular to the square plane and goes through its center.

2. *(4 points)* An equilateral triangle with a side length a, the plane of which is placed horizontally, is uniformly charged with a surface charge density of σ. A small cube with a charge q can glide without friction along the axis of the symmetry of the triangle that is perpendicular to its plane. At the equilibrium (in the presence of the gravitational field g) the cube is located at the point A on the distance $L = a/\sqrt{2}$ from each of the vertices of the triangle.

   (a) *(2 points)* Find the mass m of the cube.

   Cube is moved to the distance $r \ll a$ from the equilibrium position and released with no initial velocity.

   (b) *(2 points)* Find the velocity when it is passing through the equilibrium position.

3. *(3 points)* At the center of the square with a side $2L$, there is a ball with a mass m. Four springs connect the ball with the centers of the square sides. All springs have a tension of k. The ball is moved from the equilibrium position for the distance $a \ll L$ in arbitrary direction. Find the relation between its coordinates and time.