# The method of images

## Theory

Consider the following problem. Find the electric field strength at any point in the space $A$ for a system of charges and conducting bodies with a known potential (see the figure).

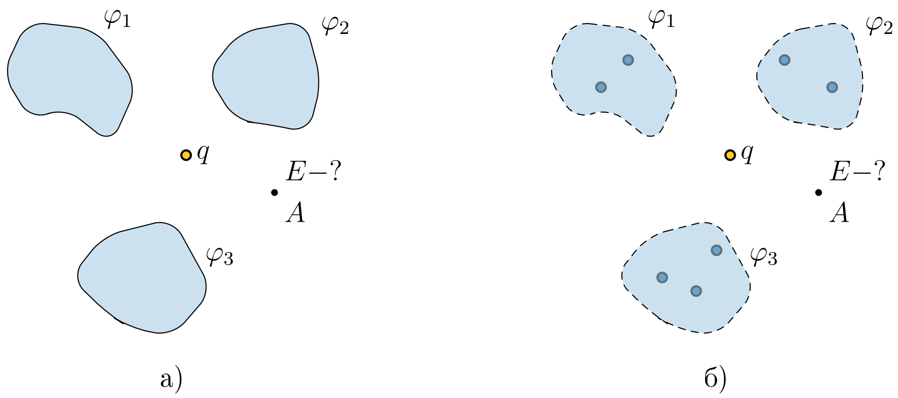



Рис. 1: The figure on the left is the original problem, and the figure on the right is an equivalent problem, with image charges instead of conductors.

The problem seems quite complicated because charges, which are induced on the surfaces of the conductors, contribute to the electric field strength. And the charge distribution is unknown.

When solving this type of problem, the *image method* is quite effective. The essence of the method is shown in the figure and consists of the following sequence of actions:

1. We find a system of charge-images that are located *inside* the conductors so that the total potential from the charge-images and the original charge system on the surfaces of the conductors is exactly the same as in terms of the problem.
2. Then we can find the electric field strength at any point *outside* of the conductor as a superposition of the electric field strengths from the original system of charges and image charges. The result will be exactly the same as in the original problem.

We will not prove this method, but we will make a few comments.

**Note 1.** The energy of interaction of the charge with induced charges on the surface of the conductor equals to the energy interaction of the charge with image charges.

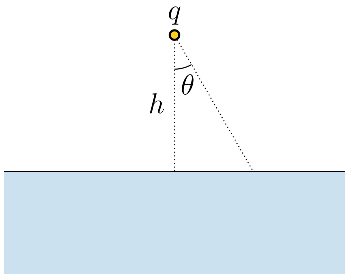
**Note 2.** this method is usually used for spherical surfaces.

We suggest you to solve several classic problems as «zero problems».

## Surfaces

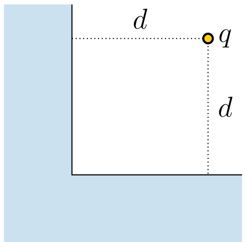
*(0 points)* You do not need to send solutions to the problems in this section!

1. The point charge $q$ is located above the conducting half-space at distance $h$. Consider a system of image charges for this problem. Find the force acting on the point charge from the induced charges. Find the value of the electric field strength on the surface of the conductor, and, knowing it, find the dependence of the surface charge density on the angle $\theta$.

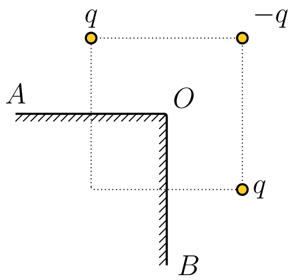



*Answer.* $F = k\frac{q^2}{4h^2}$, $E(\theta) = k\frac{q}{h^2}2\cos^3\theta$, $\sigma(\theta) = \frac{q}{h^2}\frac{\cos^3\theta}{\pi}$.

2. Make an image system and find the force acting on the point charge from the induced charges.



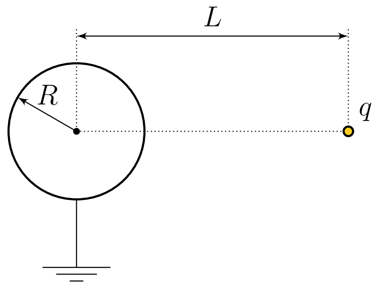

3. Three different point charges are located in the tops of a square with a diagonal length $L = 50$ cm, as shown in the figure, where $O$ is the center of the square, $AOB$ is a right angle formed by two conducting surfaces. Find the force acting on the charge $-q$ if $q = 11\ \mu C$.



## Spheres

*(0 points)* You do not need to send solutions to the problems in this section!

4. The charge $q$ is located at a distance $L > R$ from the center of the grounded sphere with radius $R$. Using the fact, that potential of grounded sphere equals to zero, find the value and position of the image charge. What is the force interaction of a point charge with charges induced on the surface of the sphere




5. Prove that the potential of a charged conducting ball with radius $R$ with a charge $Q$ in the outer field is

$$\varphi_0 + kQ/R,$$

where $\varphi_0$ is the potential of the outer field in the center of the ball. Using this result, find what the potential of such a conducting ball in the field of a point charge $q$ located at a distance $L > R$ from the center of the ball.

6. The point charge $q$ is located at a distance $L$ from the center of the conducting sphere with radius $R < L$ with a charge $Q$. What is the force of interaction of a point charge and charges on the surface of the ball? Make a plot of force acting on the point charge from the induced charges on the distance $L$ for the cases $Q/q = 3$, $Q/q = 1$, $Q/q = 0$, $Q/q = -1$. We recommend measuring the distance $L$ in the radius of the ball, and the force value in $kq^2/L^2$.

**IMPORTANT!** The next task is both a hint and an alternative to the main task. Three important points:

(a) You can continue to send the solution to the main problem.

(b) at any time before the final deadline, you can switch to *alternative task*. If you do this, write *a*t the very beginning of the solutionI'm moving on to an alternative task!. In this case, you get an additional coefficient of 0.7, which is multiplied by the old coefficient, and the solutions to the main problem are not checked from this point on. Be careful!

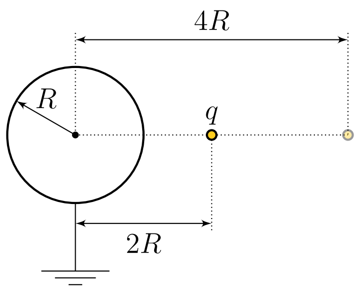
(c) the Task consists of several items. The penalty multiplier earned by **before** is applied to all points. In the future, each item is evaluated as a separate task. If you send a solution without any item, this item's solution is considered as Incorrect. For more information about scoring points for composite tasks, see the rules of the Cup.

## Alternative Problem. Energy

**Part 1** *(1 point)* Find the energy interaction of the charges on the surface of a conducting sphere with radius $R$ with a charge $Q$.

**Part 2** *(2 points)* A point charge is located at a distance of $L$ from the conducting half-surface. Find the interaction energy of a point charge with induced charges. The charge is slowly moved to a distance of $2L$ in a straight line from the conducting half-surface. Using the definition, integrate directly the work differential, find the work of moving the charge. Using this result, find the change in the interaction energy of the induced charges on the surface of the conductor with the charge.

**Part 3** *(5 points)* At a distance of $2R$ from the center of a grounded conducting sphere of radius $R$, there is a point charge $q$. The charge is moved to a distance of $4R$ from the center of the ball. What is the work of moving the point charge $q$? What is the difference between the energy of interaction of induced charges?




**Part 4** *(2 points)* Solve the previous problem for the case of an ungrounded sphere with a charge $Q$.