## Hint 2

**IMPORTANT!** The next task is both a hint and an alternative to the main task. Three important points:

1. You can continue to send the solution to the main problem.
2. At any moment before the final deadline you can start to solve the Alternative problem. If you do so, at the beginning of the solution write: *I am doing the Alternative problem!* In this case a penalty coefficient for the Alternative problem is
$$0{,}7 \cdot \sum_i \frac{k_i \cdot p_i}{10},$$
where $p_i$ is a point for the problem item, and $k_i$ is a penalty coefficient for the corresponding problem's item at the moment of moving to the Alternative problem. In other words, maximal points for the alternative problem equals to the maximal points you can gain at the moment of moving to the alternative one multiplied by 0,7. Also, we remind you that a penalty coefficient can't be less than 0,1. Solutions of the main problems from that moment will not be checked. Be careful!
3. The task consists of several items. The penalty multiplier earned by **before** is applied to all points. In the future, each item is evaluated as a separate task. If you send a solution without any item, this item's solution is considered as Incorrect. For more information about scoring points for composite tasks, see the rules of the Cup.

## Theoretical help

Let's answer the question from the main problem about the physical meaning of situations when one of the coefficients of the $ABCD$ matrix is equal to zero. We remind you that the ray transformation equation has the form of
$$\begin{pmatrix} y_2 \\ v_2 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} y_1 \\ v_1 \end{pmatrix}.$$

**1.** Consider the case $D = 0$. Consider the corresponding line of the system of equations which has zero
$$v_2 = Cy_1.$$
According to the equation, all rays leaving the same point $y_1$ will go from the optical system at the same angle $v_2 = Cy_1$ no matter what angle these rays hit it. Hence, the input plane of the optical system is the focal plane.

**2.** Consider the case $B = 0$. The first line of the system of equations will have the form of

$$y_2 = Ay_1.$$

That means that all rays outgoing from point $O$ with coordinate $y_1$ will pass through the same point of the output plane with coordinate $y_2$. In other words, the input and the output planes of the optical system are conjugate (i.e. one plane contains the source and the other contains its image). Coefficient $A$ in this case is the linear magnification factor. If the coefficient is greater than zero, the image is direct; if the coefficient is less than zero, then the image is inverted.

**3.** Consider the case $C = 0$. In this case, we get

$$v_2 = Dv_1.$$

In this case, the parallel beam remains parallel when enters the optical system. This lens system is usually called *afocal* or *telescopic*. Factor $D$ is called the angular magnification factor.

**4.** Consider the case $A = 0$. The corresponding equation takes the form of

$$y_2 = Bv_1.$$

This means that rays entering the optical system at the same angle pass through the same point with coordinate $y_2$ in the output plane. Therefore, that the output plane is the focal plane.

# Alternative problem

1. *(2,5 points)* A parallel beam of light passes through a transparent spherical ball with a diameter of 2 cm made of organic glass. The refractive index of the ball is 1,4. At what point behind the ball will the light come into focus? Additionally, consider the case with the refractive index of 2,0.
2. *(2,5 points)* An object with the size of 5 cm is located at a distance of 3 m from the screen. What should be the focal length of the lens and where the lens should be placed to give to make the object image on the screen had a size of 100 cm?
3. *(2,5 points)* The eyepiece of the Hedgehog telescope consists of two thin positive lenses with optical powers $P_1$ and $P_2$ made of the same material and located at some distance from each other. At what distance between lenses a dependence of the refractive index of glass on wavelengths will not affect the optical power of the eyepiece? Consider the wavelength being placed in a small spectral interval in the surrounding area of a wavelength $\lambda_0$.
4. *(2,5 points)* A ray of light enters from the left into a glass sphere of radius $r$. The refractive index of the glass is $n$. When it reaches the right boundary surface of the ball, some of its rays are reflected back and again appear on the left side of the ball. Find the transformation matrix of the rays for this case, and as a reference plane, you should take the plane adjacent to the left of the surface of the ball. **Note.** Theory about the reflection matrix you can find in the main task of 11 grade.

**Note.** For all tasks, consider the refractive index of the external environment equal to one.